VIBRATIONAL RESPONSE OF STRUCTURES DUE TO EXTERNAL DYNAMIC LOADING

IN PARTICULAR: INVESTIGATIONS OF THE BEHAVIOUR OF CROSS BRACED WATER TOWERS IN EARTHQUAKES

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by

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ABSTRACT

A method of predicting the seismic response of cross braced elevated water tower structures, using an electronic digital computer to integrate the equations of motion of mathematical models of the system, is presented.

The validity of the models is established by experiments on a prototype water tower, which serve to verify the allowances made for the convective action of the water, and by tests on a simple braced frame, from which it is concluded that selected multiphase lateral stiffness characteristics can be achieved with satisfactory reliability.

The earthquake response of a typical elevated water tank, with braces having various elastic and post yield properties, is determined and conclusions are drawn regarding the desirable characteristics of this form of structure if it is to resist earthquakes effectively.

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LIST OF SYMBOLS

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A	integration constant or cross sectional area
$\{A\}$	vector used in numerical integration
B	integration constant
$\{B\}$	vector used in numerical integration
C	integration constant
$[C]$	damping matrix
c_r	damping coefficient
c_g	damping coefficient
$[D]$	matrix of pivots
d	displacement
d_1	column outside diameter
d_2	column inside diameter
d_{yb}	displacement to yield bracing
$\{d\}$	vector of distortions
$\ddot{d}$	acceleration
E	elastic modulus
e	base of Naperian logarithms
F	lateral force
$\{F\}$	vector of member forces
g	acceleration due to gravity
h	height of liquid in tank
h_1	height at which m_1 is considered to act

h_o height at which m_o is considered to act

h_{so} height of centre of mass position

I second moment of area of section

$[K]$ stiffness matrix

$[K^*]$ matrix used in numerical integration

$[K]_{LAT}$ lateral stiffness matrix

K_o strength coefficient

$\{k\}$ element stiffness matrix

k_1 effective spring stiffness of convective mass m_1

k_o lateral stiffness of tower structure

k_{so} lateral stiffness of mass m_{so}

L length

$\{L\}$ vector of applied loads

$[M]$ mass matrix

m total mass of liquid

m_1 mass of liquid acting convectively

m_o mass of liquid acting with the structure

m_s mass of structure

m_{so} sum of m_o and m_s

n strain hardening coefficient

n (subscript) mode number

$[Q]$ multiplier matrix

$[Q^T]$ transform of $[Q]$

R tank radius

s engineering strain

t time

t'	particular time
x	displacement
$\{x\}$	vector of mass displacements
x_1	displacement of mass m_1
x_{so}	displacement of mass m_{so}
$\dot{x}$	velocity
$\{\dot{x}\}$	vector of velocities
$\dot{x}_t$	velocity at time t
$\dot{x}_{t-\Delta t}$	velocity at time $t-\Delta t$
$\ddot{x}$	acceleration
$\{\ddot{x}\}$	vector of accelerations
$\ddot{x}_g$	base acceleration
$\ddot{x}_t$	acceleration at time t
$\ddot{x}_{t-\Delta t}$	acceleration at time $t-\Delta t$
$\{y\}$	vector of displacements
$\{\Delta R\}$	vector used in numerical integration
Δt	increment of time
$\{\Delta x\}$	vector of incremental displacements
$\{\Delta \dot{x}\}$	vector of incremental velocities
$\{\Delta \ddot{x}\}$	vector of incremental accelerations
ϵ_1	true strain
σ_1	true stress
σL	change in length
θ	angular distortion
ω	natural frequency of vibration

CHAPTER ONE

INTRODUCTION

The problem of designing structures to minimise the effect of earthquakes has prompted extensive research in structural dynamics. The prediction of the elastic response of a structure to prescribed time varying loads or ground movements may readily be undertaken using the principles of general vibration theory[1 & 2]. However seismic disturbances are invariably unique and structures subjected to major earthquakes rarely behave in a linearly elastic manner. Thus the design problem is complicated by the indefinite nature of the disturbance and the complex behaviour of the structure.

Much attention has been devoted to the design of earthquake resistant multistorey framed structures and current earthquake resistant design philosophy involves the acceptance of two behaviour concepts. Buildings are assumed to resist small and medium seismic disturbances elastically, whereas plastic deformation is considered to provide collapse resistance to the near-catastrophic loading associated with a major earthquake[3].

For the purposes of design it is usual to simulate dynamically induced earthquake loads by equivalent static forces. Current practice involves the calculation of the dynamic elastic properties of a proposed structure and the application of the response spectra, normal mode technique to the determination of the design loads[4,5,6].

Allowance for the inelastic nature of the response at near-catastrophic loads is made by reducing the ordinates of the elastic response curves when deriving design spectra[2,7,8].

So far as it has been possible to ascertain the procedure outlined above has been applied successfully to the seismic design of multi-storey framed structures.

However there are certain types of structure to which the method cannot be extended with impunity. A water tower is one such structure. Typically a braced steel framework is used to support a rectangular or cylindrical liquid storage tank and recognition that this form of structure differs substantially in its seismic response from multi-storey, multibay framed buildings is implied in earthquake resistant design codes[9]. The specified seismic design coefficients for water towers are of the order of twice those listed for buildings but this only partly reflects the development of a rational design procedure. The provisions are, in fact, somewhat arbitrary and progress towards a better justified design approach awaits an improved appreciation of the dynamic interaction of the elements comprising a water tower and the behaviour of the whole structure under earthquake loading.

Some thirty years ago considerable effort was expended in investigating the seismic behaviour of water towers. Jacobson[10] used a large shaking table for experiments on model towers and similar work at M.I.T.[11] emphasized the necessity for a dynamic design approach to be adopted.

Westergaard's work[12] on the changes of water pressure arising on dams due to earthquake motions was incorporated with the theory

developed from model considerations by Hoskings and Jacobson[13] who found satisfactory agreement with theoretical results when the effective water mass acting in an impulsively loaded rectangular tank was measured. Carder[14] tested steel water tanks by a series of pull back tests on the supporting towers and thereby studied the influence of foundation conditions, bracing properties and loading characteristics on the manner in which steel water towers vibrated with the object of determining their likely behaviour in an earthquake. An appreciation of the effect of initial tension in cross bracing rods were shown in the work of McLean and Moore[15]. They pointed out that in a laterally loaded three panel tower, having similar bracing in each panel, the upper tie rods buckled first, then the second panel rods and finally the diagonals in the lowest panel. A tower may pass through all stages in half a cycle of motion and as a result the period may be a composite of the four computed periods. Williams[16] described investigations of the elastic behaviour of a cross braced elevated water tank and showed that for a single mass structure in near resonant conditions critical displacements may be built up before the type of ground motion changes.

Extensive model studies were undertaken by Ruge[17] with the object of deriving data useful in the designing of seismically loaded water tanks. The concept of sacrificial cross bracing was discussed and he proposed the incorporation of specially designed spring elements in the structure, however only qualitative reference to damping and inelastic action was made. Ruge drew attention to the advantages gained by progressive rather than simultaneous failure of the

structural elements. Murphy[18] carried out tests on an 84 ft. cross braced steel water tower at the Woburn Railway workshops at a result of which the natural period of vibration of the structure was determined. He suggested that the damage caused to the tower in the 1942 Wellington earthquake was consistent with that predictable using a Californian type response curve. Ulrich and Carder[19] discussed the increase in stiffness and decrease in damping observed in tests of a cross braced water tower when loose tie rods had been tightened. They submitted that their observations supported the idea that if a structure becomes damaged by a strong earthquake the period will probably increase because of the loss of rigidity and as the elastic resilience is partly destroyed the internal damping will be somewhat higher and this serve to protect the structure, partly offsetting its damaged condition. The behaviour of typical elevated water tanks in the 1952 Californian earthquake was described by Steinbrugge and Moran[20]. They reported stretching of anchor bolts and bracing rods and the complete collapse of some tanks. They deduced the modes of failure and inferred that torsional motion had occurred in some cases. Steinbrugge and Moran observed that wind braced tanks had not behaved satisfactorily whereas those designed for seismic loading had behaved better. They concluded that the currently effective design coefficients were not too large and that more study of the seismic response of cross braced elevated water towers was needed. After analysing the behaviour of water tank structures in the 1952 Californian earthquake Moran and Cheney[21] reported satisfactory correlation between the behaviour

predicted using the response spectrum approach and the actual damage sustained.

Steinbrugge and Flores[22] described the stretching of cross bracing in elevated water tank towers resulting from the 1960 Chilean earthquake and Clough and Jenschke[23] carried out computer analyses of cross braced frames based on the steel frame buildings at the University of Concepcion in which diagonal bracing was incorporated. After this was broken the unbraced structure resisted the earthquakes occurring on the following day with no further structural damage thus giving rise to the question of whether the bracing was either necessary or beneficial. A digital computer study was made of the strengthening effect of the diagonal bracing when the buildings were subjected to ground motions of three different earthquakes. Results indicated that for some earthquakes the bracing may be beneficial whereas in other cases the bracing induced forces in the frame which exceed the strength of the braced structure. Thus the bracing may actually result in the structure being less earthquake resistant than if it was unbraced. Housner[24] has proposed a method of limit design based on the energy input by recorded strong ground motions. The velocity response spectra concept was used to define the energy fed into a structure and part of this energy was assumed to be dissipated by yielding of the structure. Housner showed that a cross braced tower designed to resist, at the limit of its elastic range, an equivalent static load of 0.125 g will be safe in a disturbance which would produce 0.40 g in an elastic structure, providing that the bracing rods may be permitted to stretch.

A method of determining the hydrodynamic pressures developed in a fluid container subjected to horizontal accelerations has also been proposed by Housner[25]. The simplifications of earlier work including that of Jacobsen[26] and Graham and Rodriguez[27] involves some approximation but the technique is considered sufficiently accurate for seismic design use[28]. Both Cloud[29] and Blume[30] have reported reasonable correlation between the observed behaviour of water tanks and the vibrational characteristics predicted using Housner's method. In addition satisfactory agreement between the behaviour of petroleum storage tanks in the 1964 Alaskan earthquake and the trends expected from theoretical considerations based on Housner's work has been demonstrated[31].

However the considerable progress which has been achieved in the last decade in understanding the behaviour of buildings subjected to seismic loads has not been made in the fields of other structures such as elevated water tank towers and bridges. During this period substantial research effort expended on laboratory and site measurements, paralleled by significant theoretical investigations, has enabled much to be learnt of the dynamic characteristics of buildings in both the elastic and inelastic ranges. A more realistic approach towards the solution of the earthquake design problem for buildings is consequently being developed. The large proportion of papers describing investigations of building problems, in comparison with the handful of papers relating to elevated water tank towers, presented to the World Earthquake Engineering Conferences emphasises the relatively small effort being expended on these more specialised

structures.

Chandrasekaran and Krishna[32] have reported on experimental and analytical investigations of the seismic behaviour of reinforced concrete water towers. They recommended incorporation of diagonal steel braces in concrete frame towers to improve the strong motion response and suggested that a reinforced concrete elevated water tank may be analysed satisfactorily as a single degree of freedom system. This simplification was also used by Ramiah and Gupta[33] whereas Sonobe and Nishikawa[34], Infrim and Bratu[35] and Garcia[36] have advocated a two degree of freedom representation.

The computer simulation techniques which have been applied widely to building seismic response analysis in the last decade have received scant attention by those concerned with the earthquake behaviour of elevated liquid storage tanks. However Hanson and Fan[37] have used a digital computer to examine the effect of minimum cross bracing on the inelastic response of multistorey buildings and have shown that significant reductions in the ductility and energy absorption capacity requirements of the main frame members may be achieved by the provision of suitable diagonal members.

The principle of supplying load resistance by direct tension or compression members rather than by elements acting primarily in bending is not in itself new[38]. The problems which may arise due to incorrect design of a braced framework are well recognised[39,40]. Also the concept that a braced frame may ride out a strong motion earthquake by progressive weakening of the structure has been suggested previously[41]. Nevertheless an examination of the seismic

behaviour of a range of cross braced elevated water tower structures using an electronic digital computer simulation technique does not appear to have been undertaken. The feasibility of providing expendable cross bracing and of predicting the response of elevated water tower structures which incorporated such structural components appears worthy of study and this consideration prompted the investigation described in this thesis.

In Chapter Two the mass representation of a water tower system, involving allowance for the convective motion of the fluid, is described and the validity of the model is substantiated by the results of a simple dynamic test on a prestressed concrete elevated water tank on the Ilam site of the University of Canterbury.

The concept of multiphase cross bracing is discussed in Chapter Three. Experiments undertaken on a simple cross braced frame are described and from these it is concluded that selected multiphase response may be achieved reliably in cross braced frames if sufficient attention is given to the determination of the material characteristics and to the structural detailing.

The seismic analysis of structures using digital computer mathematical modelling techniques is outlined in Chapter Four and, in particular, the calculation of the earthquake response of a yielding system by numerical integration of the appropriate equations of motion is described. The application of this analysis method to the determination of the displacement response of elevated water tanks supported on cross braced tower structures, to digitised seismic ground accelerations is explained.

Digital computer programs used in this analysis procedure are outlined in Chapter Five and the specially written cross braced frame dynamic analysis program XBFRME is described in detail. Results of verification checks on this program are given and some limitations of the mathematical model on which it is based are discussed.

Moran and Cheney's 100,000 (U.S.) gallon water tank supported on a three storey, single bay, cross braced frame, 105 ft. high, is used as the basis for the series of seismic response predictions described in Chapter Six. Differences in the response are determined between the situation in which entirely elastic response is assumed, and those in which multiphase response of the bracing elements is taken into account. Also the effects are examined of varying the levels of prestress and changing other aspects of the bracing system.

In Chapter Seven the investigation described in this thesis is reviewed and certain suggestions for the improved design of seismic resistant cross braced elevated water tank structures are presented.

The Appendices include descriptions of the calculations undertaken in modelling both the Ilam and the Moran and Cheney water towers, details of the geometric and structural properties of the test frames and lists of the library digital computer programs used in the analyses.

C H A P T E R T W O

TWO MASS REPRESENTATION OF AN ELEVATED WATER TANK

II.1 Introduction

A typical elevated water tank is shown diagrammatically in Figure 2.1. To expedite computer simulation studies it is convenient to initially invoke the two degree of freedom representation[35,36] and to replace the contents of the tank by an equivalent sprung mass m_1, corresponding to the sloshing part of the liquid, and a second component m_o corresponding to that portion of the tank contents which moves with the structure, the mass of which is designated by m_s in Figure 2.2.

The effective spring stiffness k_1 may be determined from considerations of the geometric properties of the tank[28] and the tower stiffness k_o may be derived from the tower properties using standard structural analysis techniques.

The opportunity was taken to verify this two mass representation of a water tower system by comparing the predicted and measured dynamic characteristics of a prestressed concrete water tower on the Ilam site of the University of Canterbury.

II.2 The Elevated Water Tower

The structure examined is a prestressed concrete tubular tower supporting a cylindrical reinforced concrete tank and is shown in

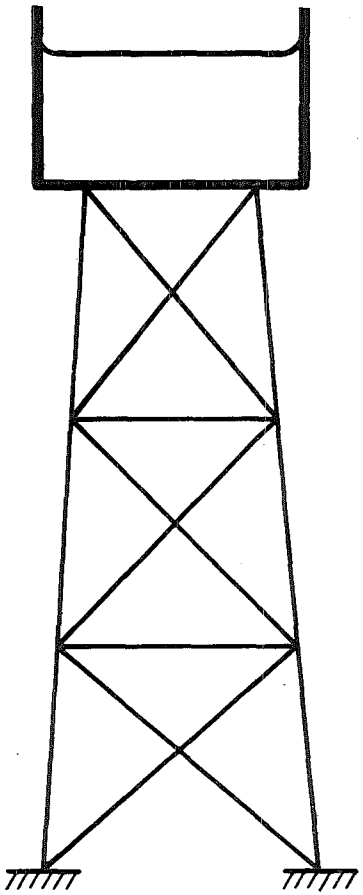


Fig.2.1: TYPICAL ELEVATED WATER TOWER

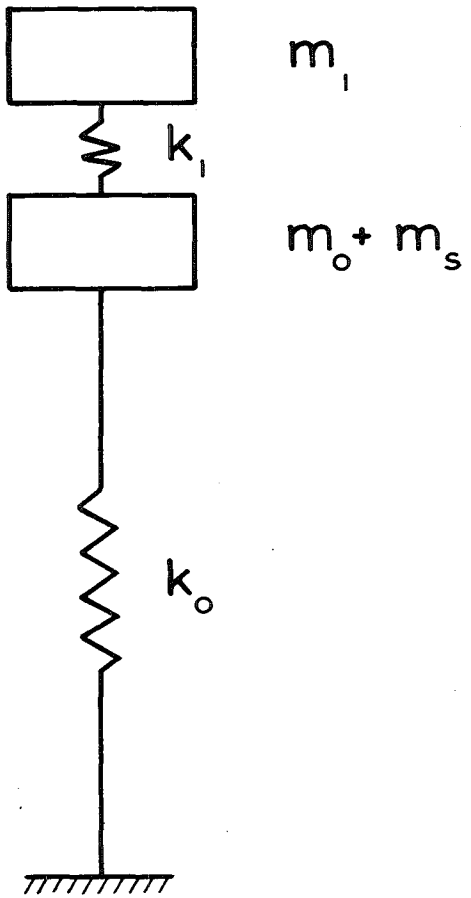
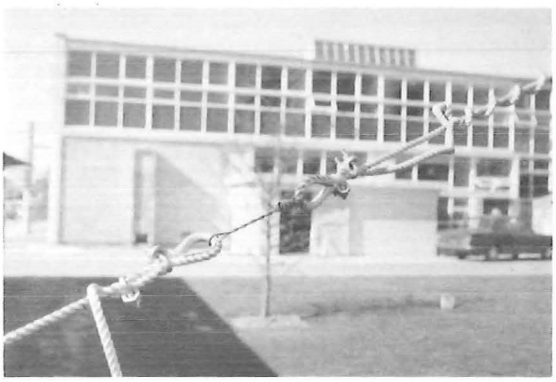


Fig. 2.2 : TWO MASS REPRESENTATION

Fig. 2.3-Ilam water tower.

(i)



(ii)

(iii)

(iv)

Fig. 2.4 — Pull back testing of Ilam water tower.

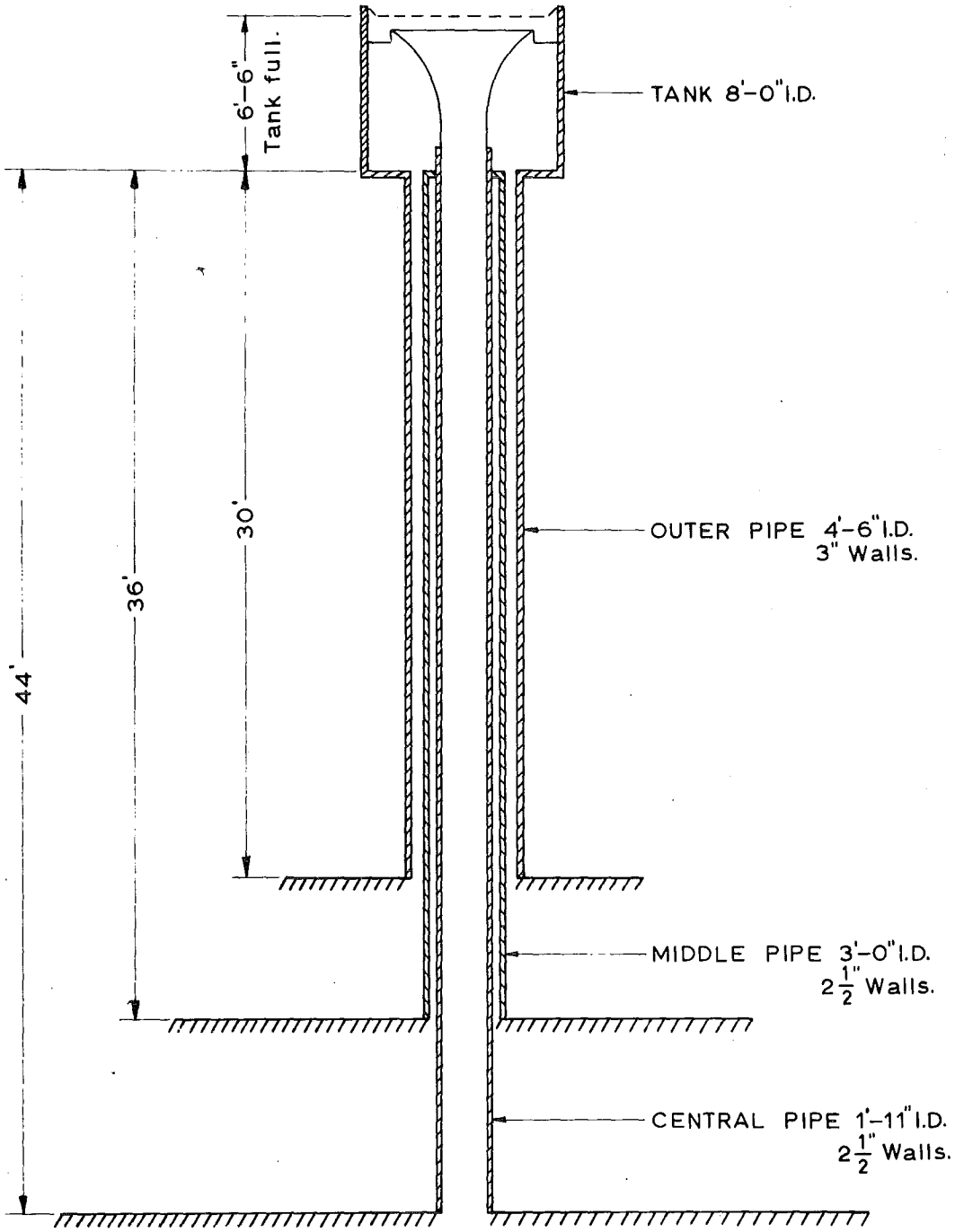


Fig. 2.5 : SECTION OF ILAM WATER TOWER

Figure 2.3. As is indicated in Figure 2.5 two concentric internal tubes form an integral part of the structure and consequently will assist in resisting lateral loads. Under normal operating conditions the tank serves to provide a constant head supply of water and the volume between the tubes is full of water.

II.3 Predicted Properties of the Ilam Water Tower

The dynamic characteristics of the elevated water tower were calculated using Housner's method[25,28]. The method of determining the hydrodynamic behaviour of the liquid container involves some approximation but the results are considered sufficiently accurate for seismic design use.

The fluid is separated into impulsive and convective parts. Then impulsive pressures are those associated with the forces of inertia produced by impulsive movements of the walls of the container, the pressures developed being directly proportional to the acceleration of the container walls. Convective pressures are those produced by the oscillation of the fluid and are thus the consequences of the impulsive pressures.

The equivalent dynamic system for a liquid storage tank is shown in Figure 2.6. The tank with an oscillating liquid surface is shown in Figure 2.6(a). The equivalent system is shown in Figure 2.6(b) where m_o and m_1 produce dynamic forces equivalent to those produced by the liquid.

The mass m_o exerts a maximum horizontal force directly proportional to the maximum acceleration of the tank bottom, at a height

h_o (see Figure 2.6(b) and thus contributes to the overturning moment in the tank.

The mass m_1 acting as a solid oscillating mass flexibly connected to the walls and located at a height h_1 also contributes to the overturning moment acting on the tank. The value of the overturning moment is computed by increasing the vertical arms to allow for the moment of the dynamic forces acting on the bottom. Thus each moment arm h_o and h_1 has two distinct values, the smaller being used to evaluate the bending moment on a plane just above the bottom, and the larger value to determine the overturning moment on a plane just below the bottom. The first case may be designated by the subscript ebp (excluding bottom pressure) and the second by the subscript ibp (including bottom pressure). Where there is a risk that the tank wall may buckle[31] the ebp moment is important. In the case of the particular concrete tank under consideration the ibp value will be of greater significance.

The mass of the tank and accessories may be accounted for by increasing m_o where necessary.

The determination of the masses m_o and m_1, the moment arms h_o and h_1 and the effective spring stiffness k_1 may be readily undertaken following Housner's work as is indicated below.

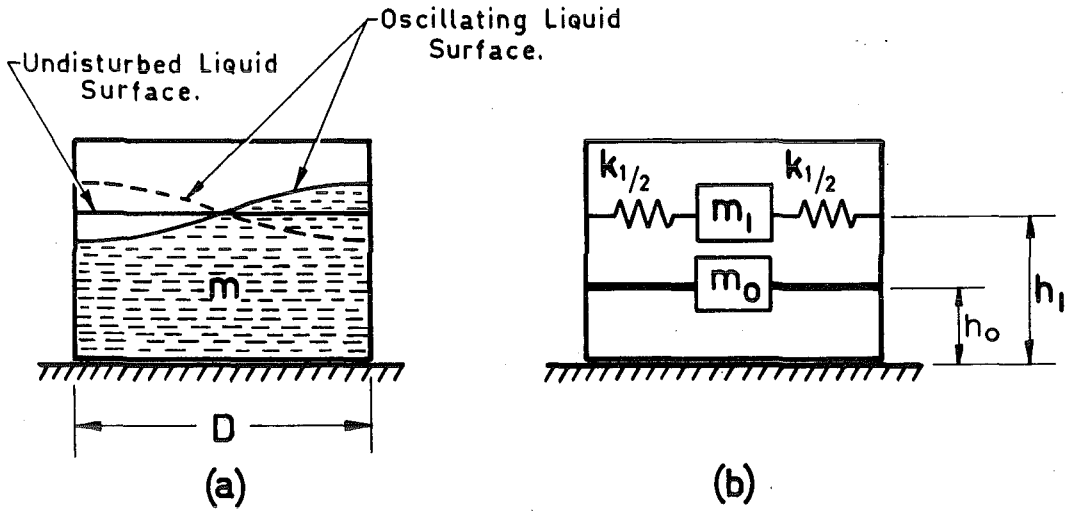


FIGURE 2.6
(After Housner)

Computation Procedure (summarised from reference 25)

Cylindrical tank case.

(a) Calculation of Impulsive Force

Steps (1) Determine the equivalent impulsive mass m_o

$$m_o = \frac{\tanh \sqrt{3}\, R/h}{\sqrt{3}\, R/h} \cdot m$$

where R is the tank radius

h is the liquid height

m is the total mass of liquid

(2) Determine the effective height h_o

$$h_o = \frac{3h}{8}\left[1 + \frac{4}{3}\left(\frac{\sqrt{3}\, R/h}{\tanh \sqrt{3}\, R/h} - 1\right)\right]$$

(3) Add m_o to the mass of the structure m_s to obtain m_{so}

(4) Determine centre of mass position h_{so}

$$h_{so} = \frac{m_o h_o + m_s h_s}{m_{so}}$$

(b) Calculation of Convective Force

(5) Determine the equivalent convective mass m_1

$$m_1 = 0.387\ R/h \cdot \tanh\ (1.84\ h/R) \cdot m$$

(6) Determine the effective height h_1

$$h_1 = h\left[1 - \frac{\cosh \sqrt{27/8}\ h/R - 135/88}{\sqrt{27/8}\ h/R \sinh \sqrt{27/8}\ h/R}\right]$$

(c) Calculation of normal mode properties

(7) Compute the lateral stiffness of the water tower i.e. the elastic load/deflection characteristic of the mass centre point (at height h_{so} above the bottom of the tank.)

(8) Determine the equivalent spring stiffness k_1

$$k_1 = 5.4 \frac{m_1^2 \, g \, h}{m \, R^2}$$

(9) Compute the normal mode frequencies and displaced shapes for the two-mass coupled system.

$$\omega^2 = \frac{1}{2}\left[\left(\frac{k_{so} + k_1}{m_{so}} + \frac{k_1}{m_1}\right) \pm \sqrt{\left(\frac{k_{so} + k_1}{m_{so}} + \frac{k_1}{m_1}\right)^2 - \frac{4k_{so}\, k_1}{m_{so} \cdot m_1}}\right]$$

where ω is the natural frequency (radians/sec)

also $$\frac{x_{so}}{x_1} = 1 - \frac{m_1 \, \omega^2}{k_1}$$

where $\frac{x_{so}}{x_1}$ is the displacement ratio of the total impulsive mass.

The computer program MODANAL (list 1) was written to expedite the computation of the modal properties of a two mass representation of an elevated water tower using Housner's equations. The input data includes geometric and mass properties of the tower and the lateral stiffness of the structure with respect to the mass centroid. The output includes the natural frequencies and associated displaced shapes of the two modes of vibration (see Figure 2.7). Typical

```
C                 PROGRAM MODANAL
C     THIS PROGRAM COMPUTES THE MODAL PROPERTIES OF A
C     TWO MASS REPRESENTATION OF AN ELEVATED WATER TOWER
C              USING HOUSNER'S EQATIONS
C
C
      REAL M,MS,KSO,MSO,K1,M1
   55 N=1
      READ(5,1,END=99) M,R,H,MS,KSO,G,HS
    1 FORMAT(7G10.0)
      RATIO=R/H
      RAT=SQRT(3.)*RATIO
      MO=TANH(RAT)/RAT*M
C     MO CALCULATED
      TRAT=3.*RATIO
      HO=.375*H*(1.+4.*(RAT/TANH(RAT)-1.)/3.)
C     HO CALCULATED
      MSO=MS+MO
C     MSO CALCULATED
      HSO=(MO*HO+MS*HS)/MSO
C     HSO CALCULATED
      M1=0.387*RATIO*TANH(1.84/RATIO)*M
C     M1 CALCULATED
      HRAT=SQRT(3.375)/RATIO
      H1=H*(1.-(COSH(HRAT)-135./88.)/(HRAT*SINH(HRAT)))
C     H1 CALCULATED
      K1=5.4*M1**2*G*H/(M*R**2)
C     K1 CALCULATED
      DISC=SQRT(((KSO+K1)/MSO+K1/M1)**2-4.*KSO*K1/(MSO*M1))
      IF(DISC.LT.0.0) GO TO 88
      CONST=(KSO+K1)/MSO+K1/M1
      WTWO1=(CONST-DISC)/2.
      WTWO2=(CONST+DISC)/2.
      IF(WTWO1.LT.0.0.OR.WTWO2.LT.0.0) GO TO 88
      XSOX11=1.-M1*WTWO1/K1
      XSOX12=1.-M1*WTWO2/K1
      W1=SQRT(WTWO1)/6.284
      W2=SQRT(WTWO2)/6.284
   44 WRITE(6,999)
      WRITE(6,2) M
      WRITE(6,3) R
      WRITE(6,4) H
      WRITE(6,5) MS
      WRITE(6,6) KSO
      WRITE(6,7) G
      WRITE(6,8) HS
    2 FORMAT(////25X,'M   =  ',E12.4,'  LBS/G'/)
    3 FORMAT(25X,'R   =  ',E12.4,'  FEET'/)
    4 FORMAT(25X,'H   =  ',E12.4,'  FEET'/)
    5 FORMAT(25X,'MS  =  ',E12.4,'  LBS/G'/)
    6 FORMAT(25X,'KSO =  ',E12.4,'  LBS/FOOT'/)
    7 FORMAT(25X,'G   =  ',E12.4,'  GRAVACCN'/)
    8 FORMAT(25X,'HS  =  ',E12.4,'  FEET'////)
      WRITE(6,10) MSO
      WRITE(6,15) HSO
      WRITE(6,20) M1
      WRITE(6,25) H1
      WRITE(6,30) K1
      GO TO (66,77),N
   66 WRITE(6,35) W1
      WRITE(6,40) XSOX11
      WRITE(6,45) W2
      WRITE(6,40) XSOX12
      GO TO 55
   77 WRITE(6,45)
      GO TO 55
   10 FORMAT(////25X,'MSO =  ',E12.4,'  LBS/G'/)
   15 FORMAT(25X,'HSO =  ',E12.4,'  FEET'/)
   20 FORMAT(25X,'M1  =  ',E12.4,'  LBS/G'/)
   25 FORMAT(25X,'H1  =  ',E12.4,'  FEET'/)
   30 FORMAT(25X,'K1  =  ',E12.4,'  LBS/FOOT'/)
   35 FORMAT(25X,'W1  =  ',E12.4,'CYCLES/SEC'/)
   40 FORMAT(25X,'XSO =  ',E12.4/)
   45 FORMAT(25X,'W2  =  ',E12.4,'CYCLES/SEC'/)
   50 FORMAT(25X,'ERROR IN CALCULATING FREQUENCY'/)
  999 FORMAT(////25X,'****************************************')
   88 N=2
      GO TO 44
   99 STOP
      END
```

TYPICAL INPUT DATA

```
 0.0      4.      0.0     700.    650000.     32.2      2.
196.      4.      2.0     700.    650000.     32.2      2.
340.      4.      3.5     700.    650000.     32.2      2.
490.      4.      5.0     700.    650000.     32.2      2.
638.      4.      6.5     700.    650000.     32.2      2.
```

LIST 1

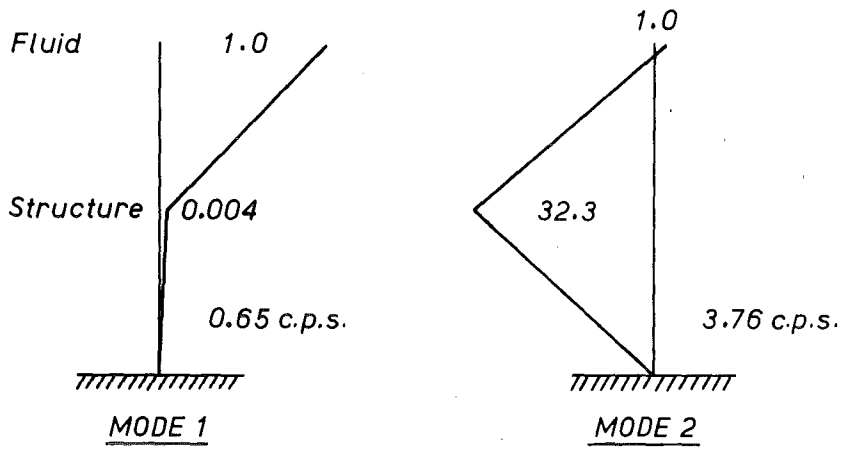


Figure 2.7: TYPICAL THEORETICAL MODE SHAPES

INPUT							OUTPUT								
FLUID MASS	TANK RADIUS	FLUID HEIGHT	STRUCT-URAL MASS	STRUCT-URAL STIFF-NESS	GRAVITY ACC $\frac{N}{}$	STRUCT-URE C.G. HEIGHT	TOTAL INERTIA MASS	INERTIA MASS HEIGHT	CONVECT-IVE MASS	CONVECT-IVE MASS HEIGHT	EFFECT-IVE UPPER SPRING STIFF-NESS	FIRST MODE FRE-QUENCY	DISPLACE-MENT RATIO	SECOND MODE FRE-QUENCY	DISPLACE-MENT RATIO
M	R	H	MS	KSO	G	HS	MSO	HSO	M1	H1	K1	W1	XSO	W2	XSO
LBS/G	FT	FT	LBS/G	KIPS/FT	FT/SEC^2	FT	LBS/G	FT	LBS/G	FT	KIPS/FT	C.P.S.		C.P.S.	
638	4.0	6.5	545	650	32.2	2.0	1016	2.7	151	4.7	2.53	0.65	0.004	4.0	-37
490	4.0	5.0	545	650	32.2	2.0	856	2.5	149	3.5	2.45	0.64	0.004	4.4	-45
340	4.0	3.5	545	650	32.2	2.0	710	2.3	139	2.5	2.16	0.62	0.003	4.8	-58
196	4.0	2.0	545	650	32.2	2.0	601	2.1	110	2.2	1.35	0.56	0.002	5.2	-88
0	4.0	0.0	545	650	32.2	2.0	545	2.0	0	0	0	0	1.0	5.5	1.0
638	4.0	6.5	700	650	32.2	2.0	1171	2.6	151	4.7	2.53	0.65	0.004	3.8	-32
490	4.0	5.0	700	650	32.2	2.0	1011	2.4	149	3.5	2.45	0.64	0.004	4.0	-38
340	4.0	3.5	700	650	32.2	2.0	865	2.2	139	2.5	2.16	0.63	0.003	4.4	-48
196	4.0	2.0	700	650	32.2	2.0	756	2.1	110	2.2	1.35	0.55	0.002	4.7	-70
0	4.0	0	700	650	32.2	2.0	700	2.0	0	0	0	0	1.0	4.8	1.0
638	4.0	6.5	800	650	32.2	2.0	1271	2.6	151	4.7	2.53	0.65	0.004	3.6	-30
490	4.0	5.0	800	650	32.2	2.0	1111	2.4	149	3.5	2.45	0.64	0.004	3.9	-35
340	4.0	3.5	800	650	32.2	2.0	965	2.2	139	2.5	2.16	0.63	0.003	4.1	-43
196	4.0	2.0	800	650	32.2	2.0	856	2.1	110	2.2	1.35	0.56	0.002	4.4	-61
0	4.0	0	800	650	32.2	2.0	800	2.0	0	0	0	0	1.0	4.5	1.0

TABLE 1 — Typical input and output data for MODANAL analysis of Ilam water tower

input and output is shown in Table 1. A summary of the preliminary calculations undertaken in the modelling of the tower shown in Figure 2.3 is presented in Appendix I and the predicted significant translation vibration frequencies of the structure, corresponding to the tank being filled with water to selected levels, are presented in Table 2.

II.4 Measured Properties of the Ilam Water Tower

A simple pull-back test was carried out on the tower. The structure was deflected laterally by applying a load immediately below the tank through a 0.5 in. diameter nylon rope anchored both to a hoist at ground level and to a steel collar fixed round the tower (Figure 2.4(i)). The rope was tensioned using the hoist and quick release was achieved by cutting a 10 gauge mild steel wire link inserted in the length (Figure 2.4(ii)). The sections on either side of the wire link were anchored to reduce the hazard arising from back lash as the link was severed. (Figure 2.4(iii)).

The motion of the tower was recorded by a horizontal displacement meter (Figure 2.4(iv)), developed for use in the small amplitude dynamic testing of multistorey buildings[42], coupled to a pen recorder.

The meter is essentially an inverted pendulum enclosed in a cylindrical housing which is sealed onto the base plate and filled with silicon oil. The relative movement between the pendulum and the housing is detected by a Philips PR 9310 inductive pickup connected to Philips PR 3304 direct reading measuring bridge which

serves to amplify the transducer signal before it is recorded on a Philips PT 2108 oscilloscript. A maximum magnification of 24,000 may be obtained using this equipment, a 1 mm division on the recording paper corresponding to a meter base displacement of 4.16×10^{-5} mm. In practice the meter was calibrated on a shaking table to establish the variation in response characteristic with both temperature and frequency.

A series of pull-back tests were undertaken, the depth of water in the tank being varied between 0 and 6.5 ft. A typical free vibration oscilloscript trace is shown in Figure 2.8(a) and a portion of a trace obtained when the tower was excited by the action of the pump which circulates water up the outer tubes of the tower to spill over the internal weir at approximately 7 cusecs is shown in Figure 2.8(b). A pronounced beat effect is evident on the second trace. The recorded frequency of 4 c.p.s. evident on both traces is that of the second mode of the system, the configuration in which the structural movement predominates (see Figure 2.7). The percentage equivalent viscous critical damping was determined from the decay of the free vibration as 1.3%.

Subsequently hand shaking tests, similar to those used in building excitation[43] were used to determine the "sloshing" frequency of the water inside the tank. Recording was undertaken in a similar manner to that used in the pull-back tests. A range of water depths were again investigated and the results are presented in Table 2.

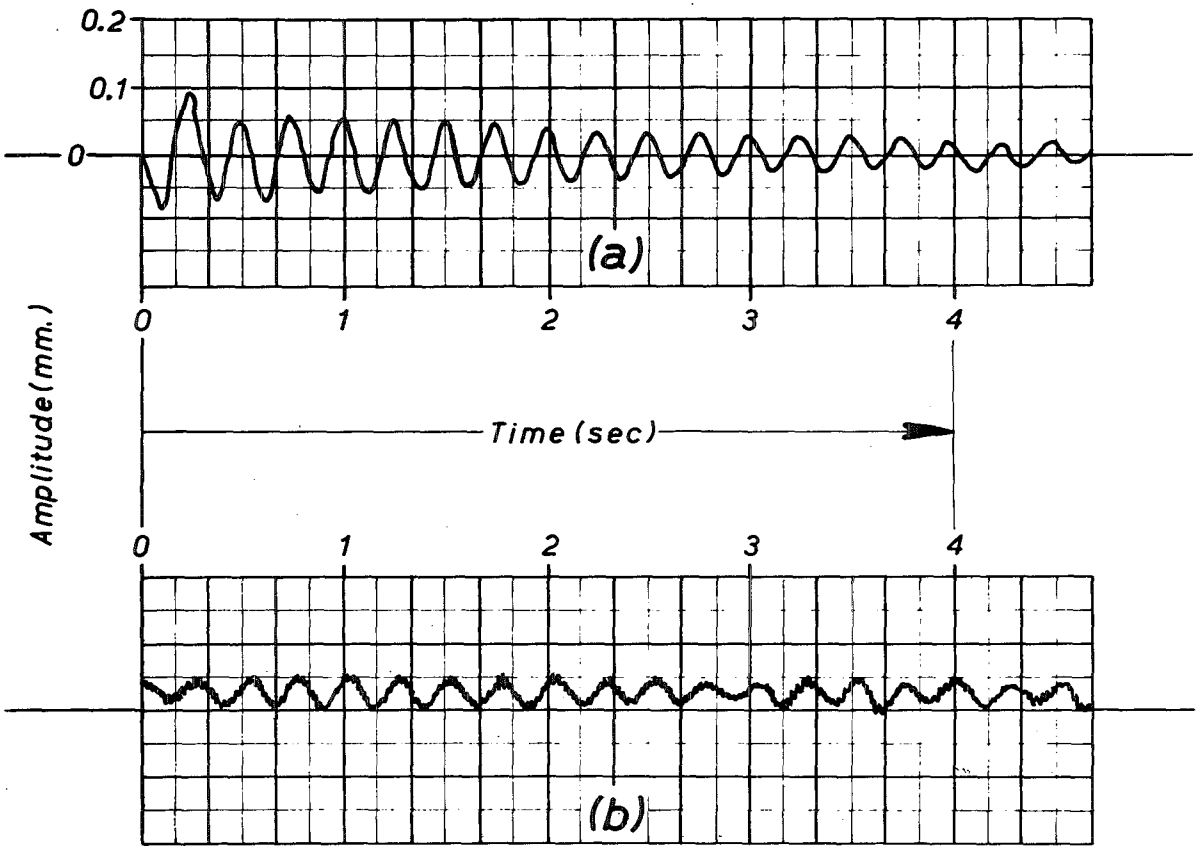


Figure 2.8: TYPICAL OSCILLOSCRIPT TRACES RECORDED ON ILAM WATER TOWER
(a) Decay Curve (b) Vibration induced by pumping

Fluid Depth	MODE 1		MODE 2			
	MODANAL Values	Experimental Values	MODANAL Analysis Results For Structural Mass MS MS=545	700	800	Experimental Values
ft.	c.p.s.	c.p.s.	c.p.s.	c.p.s.	c.p.s.	c.p.s.
6.5	0.65	0.64	4.0	3.8	3.6	4.0
5.0	0.64	0.62	4.4	4.1	4.0	4.25
3.5	0.63	0.60	4.8	4.4	4.1	4.5
2.0	0.56	0.55	5.2	4.7	4.4	4.76
0	0	0	5.5	4.8	4.5	4.9

Table 2. Comparison of measured and predicted frequencies of Ilam water tank structure.

II.5 Comment on the Two Mass Modelling of Water Towers

Where the primary objective is to determine a two degree of freedom system equivalent to a particular elevated water tower it is evident that the Housner analysis method may be conveniently programmed for digital computer execution and that a program such as MODANAL enables sensitivity analyses to be readily undertaken (see the results presented in Table 1 corresponding to variations in the quantity of water in the selected tank and to changes in the effective mass of the structural system.)

The testing undertaken on the Ilam water tower indicates that the use of a two mass representation will enable satisfactory predictions of the frequency properties to be made, particularly when this is essentially a preliminary step in the modelling of the system with the object of carrying out more complex analyses as described in the following chapters. The relatively small discrepancies between the measured and calculated frequency values can be accounted for by the hindering of the convective movement of the water by the internal weir structure. This would not be present in a typical elevated water storage tank and even better correlation could reasonably be expected in more usual configurations.

C H A P T E R T H R E E

CROSS BRACING IN EARTHQUAKE RESISTANT STRUCTURES

III.1 Introduction

Current earthquake resistant design philosophy requires structures to resist minor earthquakes without damage (i.e. elastically), to resist moderate earthquakes without structural damage but with some non-structural damage and to resist major earthquakes without catastrophic collapse but with some structural damage (i.e. inelastic deformation).

The use of cross bracing to resist lateral forces is an obvious method of providing economic stiffening in a tower structure. Increased rigidity may be achieved efficiently, in terms of material usage, in otherwise light and fairly flexible frameworks. Fifty years ago, when rigidity was considered to be of paramount importance in seismic design, cross bracing was extensively used. Certainly stiff structures do not normally suffer extensive secondary damage in small or moderate earthquakes but a growing awareness that more flexible structures tended to attract less inertia load in an earthquake disturbance, coupled with the understanding of the need for ductile characteristics, prompted the use of rigid jointed unbraced frameworks for building frames to such an extent that only in relatively light-tower structures, such as those used to support elevated water tanks, has cross bracing

continued to be generally used. In the last two or three years diagonal bracing has reappeared in several notable new buildings including the Alcoa building in San Francisco's Golden Gateway Centre and the 100 storey John Hancock building in Chicago. Nevertheless, the architectural disadvantages of bracing, coupled with some problems encountered in ensuring adequate structural integrity, appear likely to limit the use of bracing to semi-industrial structures such as elevated water tank towers. For this reason in this study consideration of the application of cross bracing is restricted to a basically simple, single bay, multistorey framework supporting a liquid storage tank.

The concept of two phase seismic response is implicit in the current seismic design philosophy outlined above. This mode of behaviour can readily be encouraged in braced frameworks by correct provision of ductile (and expendable) bracing elements inserted in an otherwise elastic frame. In the case of a braced tower with essentially rigid joints, the structure may be designed to have expendable bracing which will yield in a major seismic disturbance, thereby absorbing energy and at the same time increasing the flexibility of the structure so that the residual ductile, moment resisting frame can then survive the earthquake, despite having experienced substantial lateral displacements, by virtue of its own resistance. In the case of a braced tower with essentially pinned joints the bracing may itself be of a two phase nature, possibly provided by the use of pairs of parallel braces, one of each of which is designed

to remain elastic under all anticipated loading conditions and the second one being selected with the intention that it should yield and finally break in a major earthquake.

In parallel with the analytical study of this expendable bracing concept it was considered necessary to examine certain practical aspects of incorporating cross bracing of the type envisaged in a civil engineering structure. Consequently a feasibility study was undertaken. The simplest possible cross braced panel, consisting of a two dimensional rectangular steel frame braced with diagonal members, was modelled physically and subjected to selected racking tests. The object was to establish that behaviour of the type conceived can be obtained in a practical situation without resort to assembly techniques which might prove unrealistic and to confirm that the actual load deflection characteristics correlate satisfactorily with those predicted.

III.2 Cross Braced Panel Testing

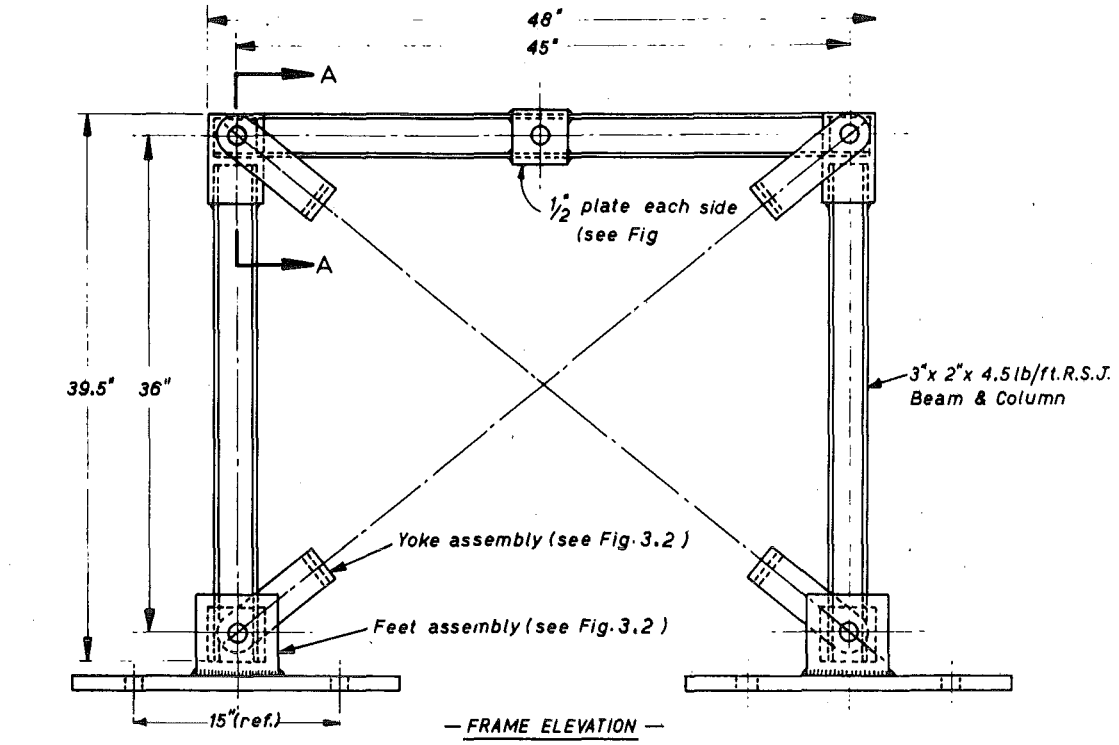
Initially a series of simple rectangular portal frames of 5 ft. 0 in. leg and beam length and of 4 in. x 2.5 in. x 6.5 lb/ft. section R.S.J., both with and without bracing, were subjected to racking loads applied through a horizontal jack bearing against the top of the leg, in line with the centre line of the beam. Secondary effects, including deformation perpendicular to the plane of the frame and local stress concentrations around the knee prevented any meaningful behaviour patterns being established but as a result of this preliminary experimental work a better controlled series of tests

was subsequently undertaken. These are described below.

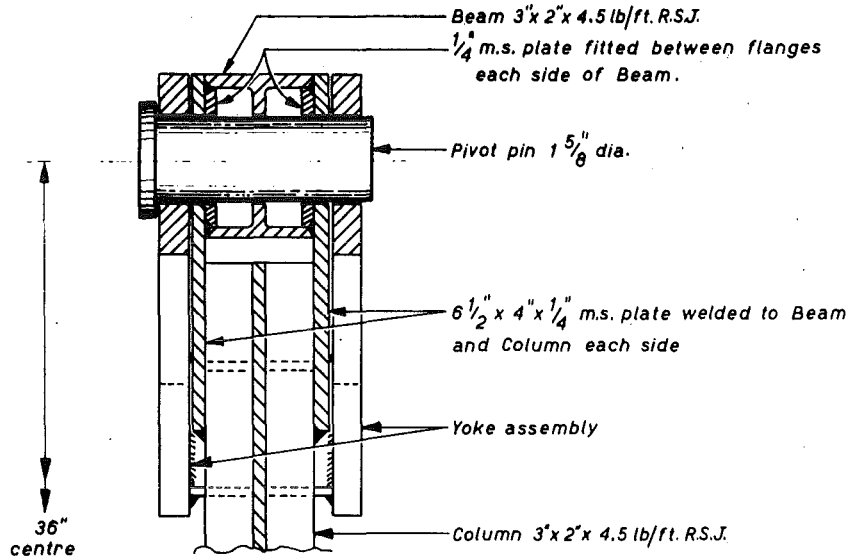
In order to facilitate laboratory testing with readily available equipment a standard braced panel fabricated from 3 in. x 2 in. x 4.5 lb/ft. R.S.J. section was selected (see Figures 3.1 and 3.2). This configuration allowed the detailing to be representative of that encountered in full scale steel frameworks without requiring particularly elaborate facilities to enable such a frame to be racked in the post-elastic range.

The span of the frame was chosen to conform to the 15 in. pitch of the holding down sockets on the test floor used and a further consideration in selecting the frame size was the availability of a variable displacement dynamic loading machine which had been constructed for, and used in, a dynamic study of structural concrete[44]. This machine is capable of applying reversed cyclic loading at frequencies between 0.5 and 5.0 c.p.s. with a maximum force output of 4 tons, a maximum throw of 3 in. (i.e. 6 in. double amplitude) but with a limitation on the torque on the main drive shaft of 4.9 ton.in. As it was intended to use this machine to cyclically load the braced panels they were proportioned accordingly.

To overcome the difficulties encountered in the preliminary testing when the racking load was applied to the face of the section at the knee position, a load link consisting of two parallel 3 in. x 0.5 in. mild steel arms, pinned to the centre of the beam, was used. (See Figures 3.3(i) and 3.3(ii)). This enabled lateral loads to be applied along the centre line of the beam section without stress concentrations being introduced at one knee position and the



— FRAME ELEVATION —



SECTION A-A

NOTE :— 1/4" welding throughout.

Figure 3.1: DETAILS OF HINGED KNEE BRACED PORTAL

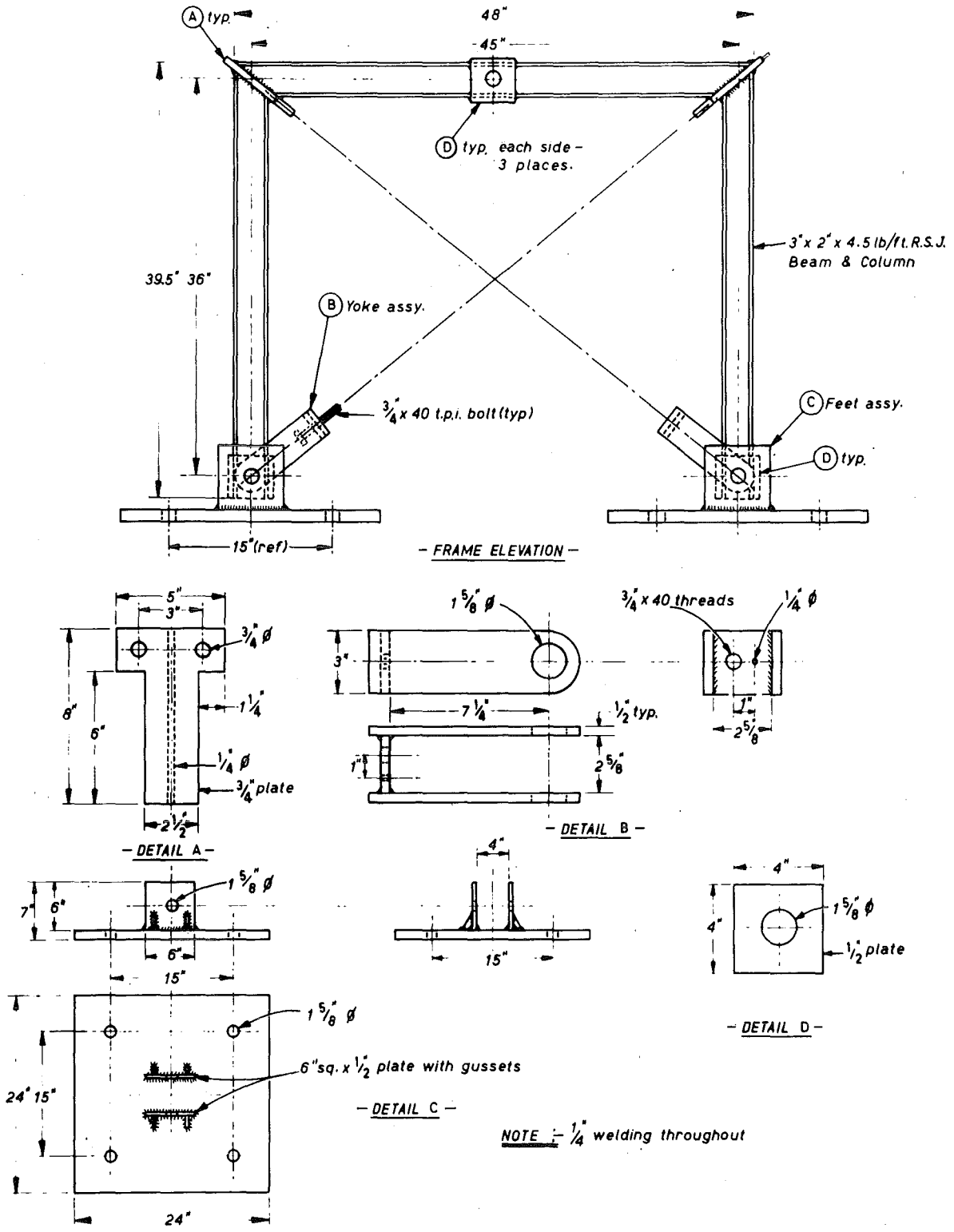


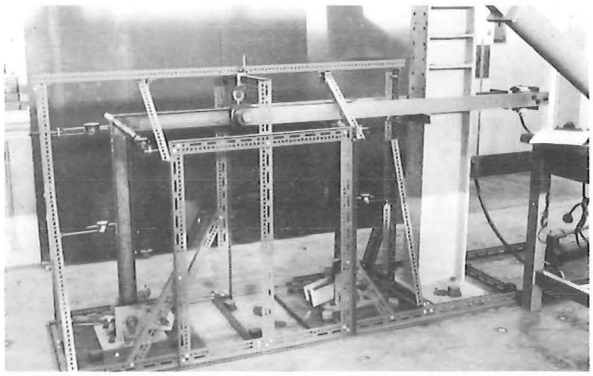
Figure 3.2: DETAILS OF RIGID KNEE BRACED FRAME

system was considered to adequately represent the inertia load situation developed in a prototype panel by earthquake movement.

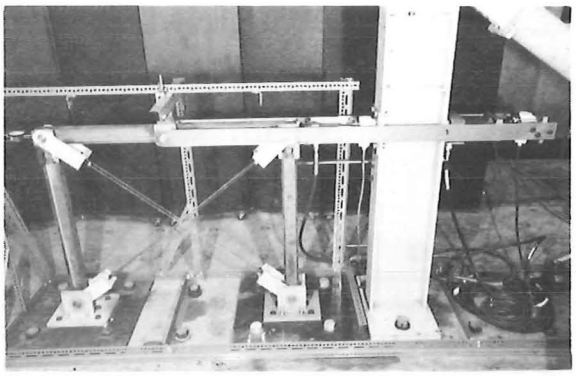
For the series of static tests undertaken provision was made to load the link in either tension or compression by means of a 10 ton Simplex hydraulic jack (model RC 106) activated by a Simplex two way hand pump. The jack reacted against a substantial laterally braced vertical cantilever (Figure 3.3(iii)) and was mounted in series with a 10 ton Philips 9266/10 load cell connected to a model 120 Budd strain gauge bridge, to enable the applied load to be determined.

When the test frames were racked by the dynamic loading machine (Figures 3.4(i) and 3.4(ii)) the load link was also used but in this case it was pinned to the machine cross head (Figure 3.4(iii)) which itself incorporates a load cell so that the transmitted axial force was again determined by means of a strain gauge bridge.

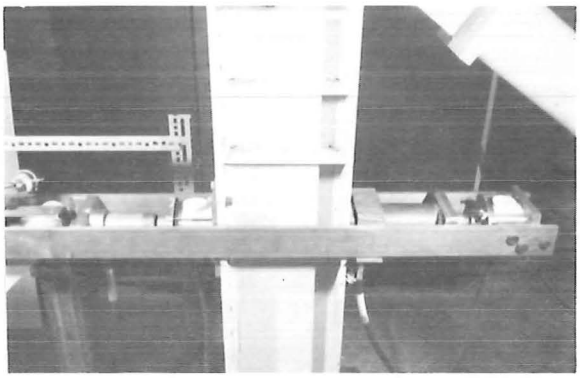
The deformation of the frame was measured by a series of 2 in. travel Mercer model Alpha 22(a) dial gauges and, in the dynamic testing, by deflection gauges consisting of strain gauged spring steel cantilever of 10 in. length and 0.5 in. x 0.0625 in. section. (See Figures 3.4(iv) and 3.4(v)). These were clamped to vertical supports bolted to the strong floor and connected to the frame under test through twin universal joints at the free ends. When recording dynamic loads and displacements, provision was made to connect the strain gauges in the load cell and the deflection gauges through Philips PR9304 bridges to a Riken Denshi X-Y Plotter Model F-3B, F-2



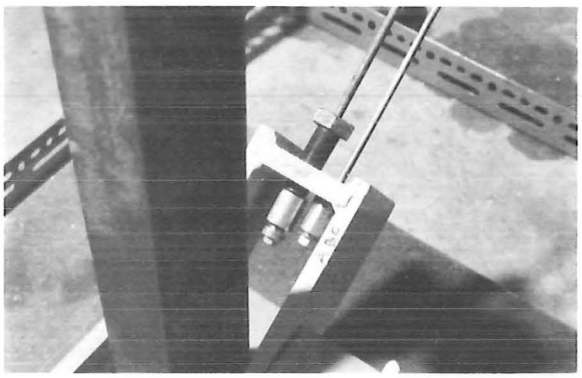
(i)



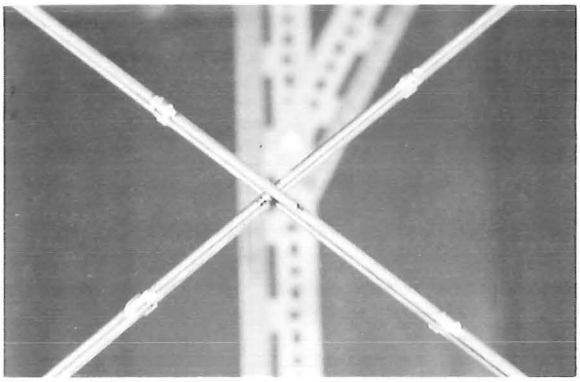
(ii)



(iii)



(iv)



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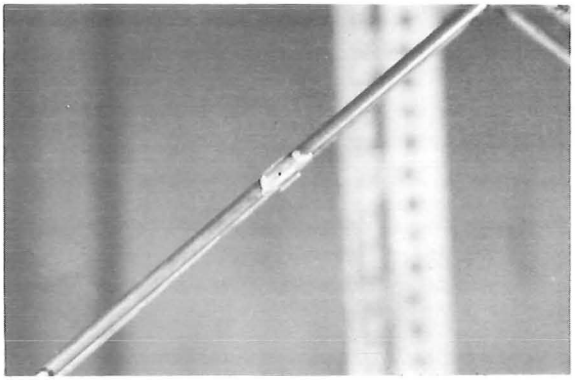
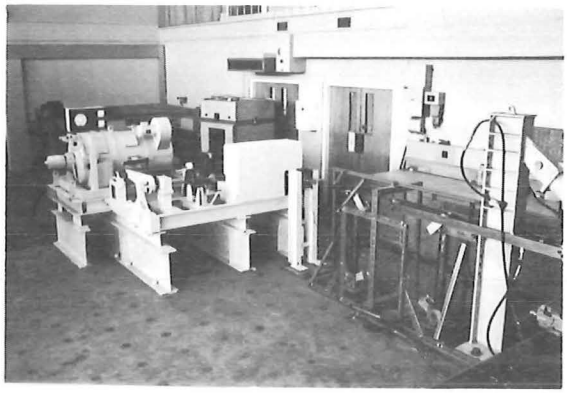


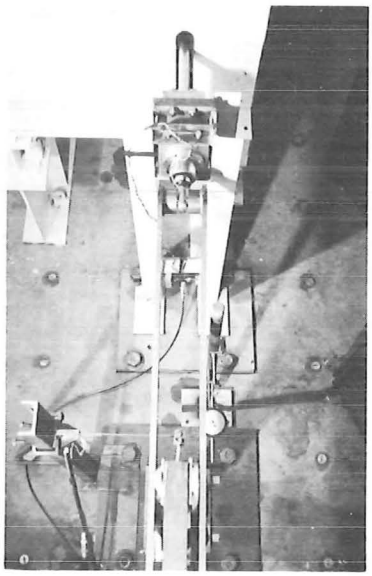
(vi)

Fig.3.3—Details of racking test rig.

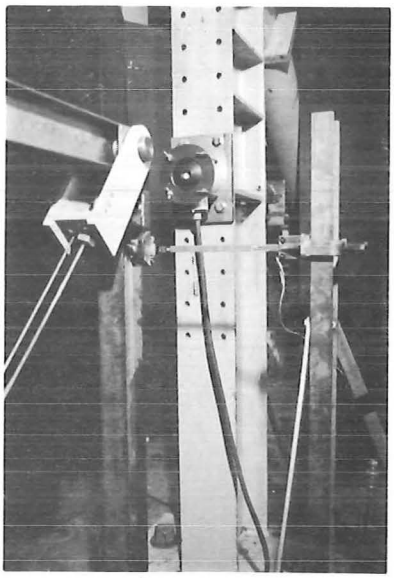


(i)

(ii)



(iii)



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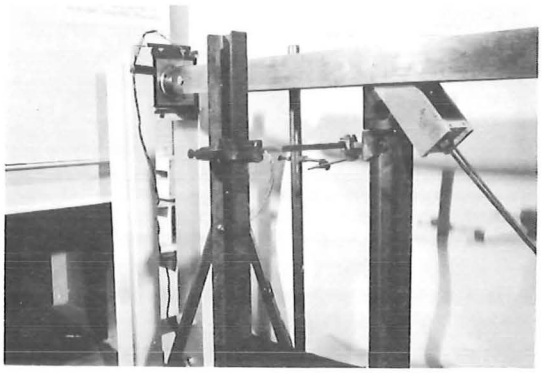

(v)

(vi)

Fig. 3.4—Details of racking test rig.

on which Load/Deflection curves were recorded. (Figure 3.4(vi)).

Two methods were used to determine the strains in the circular section bracing rods. Standard Philips PR 9814 wire resistance strain gauges were glued to the surface of the rods in pairs on diametrically opposed positions and each pair was wired in series to ensure that axial, rather than bending, distortions were measured. In addition 8 in. Demac gauges were used to determine the rod strains, the Demac points being fixed to the rods using wax (see Figures 3.3(v) and 3.3(vi)). The relative merits of these two strain measuring techniques, as applied in the tests made, are discussed below.

The bracing rods were secured in the frame in several different ways. Some of the rods used initially had plates welded to each end so that they could be bolted to the knee stiffening plate at the upper end and to the stirrup at the lower one. Other rods were anchored by passing them through holes in the stiffening plates and in the stirrups, standard prestressing anchorages being used to secure the rods. (See Figure 3.3(iv)). In the later stages of testing the hinged panels, stirrups were used at both the upper and lower ends of the bracing and in all cases a drilled adjustment bolt was used between the anchorage and the plate through which the rod passed to facilitate the seating of the grips and to enable any slack which occurred during testing to be taken up.

III.3 Material Properties

Since it was intended to load the frames and bracing both in the elastic range and beyond yield, properties of the steel elements used were established in a series of tensile tests on specimens prepared and proportioned in accordance with BS18:1962. Several rectangular cross section specimens were cut from the web of the R.S.J. section adjacent to the lengths used in fabricating the portals, and sets of circular cross section test pieces were prepared from lengths of the rods used as bracing. A 25,000 lb. capacity Avery Universal testing machine was used to apply the tensile loads and the strains were recorded using a 2 in. gauge length Baty mechanical strain gauge. Properties established included the elastic modulus, the first yield strain, the true stress/true strain post yield relationship and the ultimate load.

The need to verify the properties of the materials used was demonstrated by the first rod tested which, although supplied as "mild steel" exhibited very little ductility consistent with the material having been cold worked.

Following the work of Erasmus[45] essentially bilinear load/deflection models of axially loaded mild steel braces may be derived from consideration of true force, true deflection curves[46] and hence the material properties measured in this study were used to establish the elastic stiffness and effective post-yield elastic stiffness of the bracing elements (see Appendix II).

A typical plot of log true stress against log true strain is presented in Figure 3.5 and from this and a similar graph for the

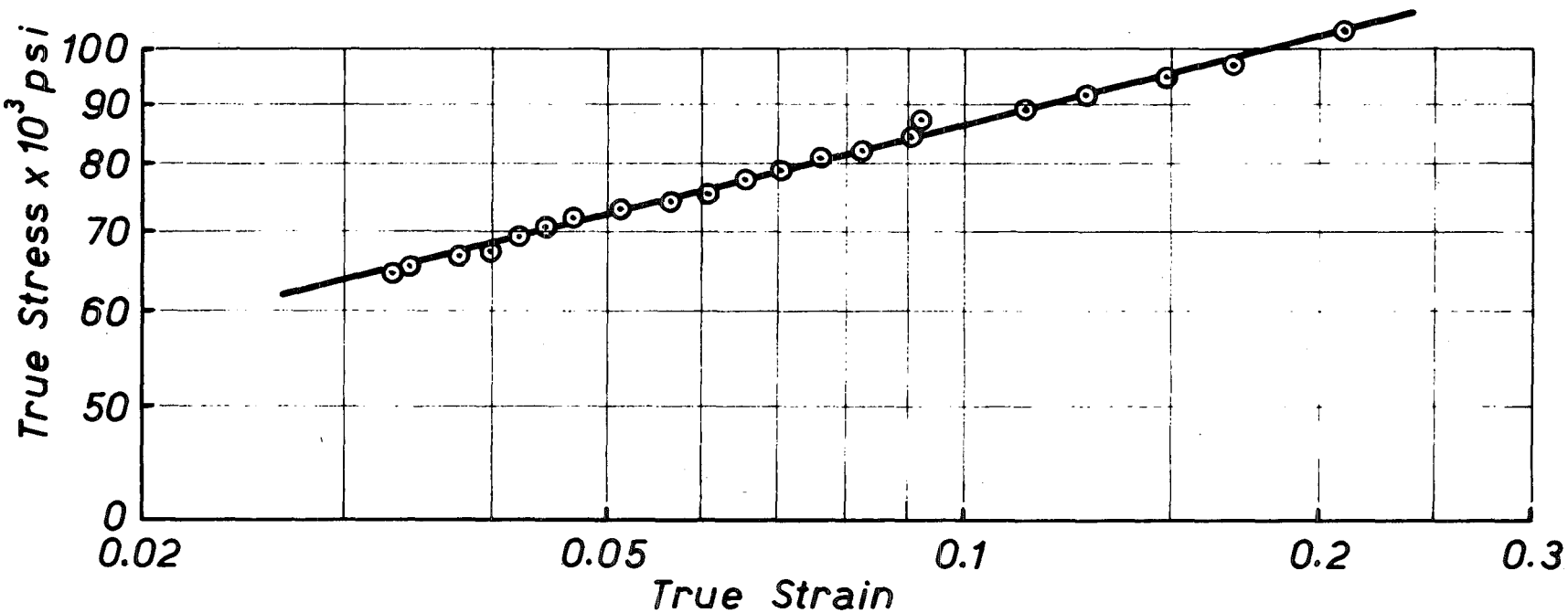

Fig.3.5 : LOG-LOG PLOT OF TRUE-STRESS, TRUE-STRAIN FOR 0.25 in. dia. MILD STEEL BRACING

thicker bracing the values of strain hardening exponent n and strength coefficient K were determined for the mild steel braces. A summary of the bracing rods' material properties is presented in Table 3. The testing of the joist material indicated that the section was of typical low carbon steel having a Young's modulus of 28×10^6 lbs/in.^{2} and a yield level of 43,500 lbs/in^2.

A simple digital computer program TRUEFORCE (list 2), based on the expression for true force developed in Appendix II, was used to calculate the post yield forces in the bracing rods for selected lateral deflections of the test portal and the results of these computations are shown in Figures 3.6 and 3.7.

III.4 Tests Undertaken

An initial static racking test was undertaken on an unbraced rigid knee portal in the rig shown in Figure 3.3(i) to establish that the difficulties experienced in the preliminary experimental work, brought about by distortion of the frame perpendicular to its own plane and by upward deflection of the base plates, had been satisfactorily overcome. This was confirmed. The jack loads were limited to those corresponding to 0.5 in deflection to avoid inelastic straining of the frame and the load/deflection plot (Figure 3.8) enabled a value of stiffness of 4,600 lb/in. to be determined. This correlates satisfactorily with the theoretical value of 4,570 lb/in. (See Appendix III).

A second static racking test on a rigid knee portal braced with one 0.25 in. diameter mild steel rod enabled the bilinear

Bracing		Young's Modulus	Elastic Limit Strain	Strain Hardening Exponents	Strength Co-efficient	Ultimate Tensile Load
Dia-meter (ins)	Type	(lbs/in^2) x 10^6		n	K_o	(lbs)
0.25	M.S.	31.2	0.00186	0.25	154,000	2,940
0.375	M.S.	29.6	0.00189	0.28	132,000	6,860
0.205	H.T.S.	28.9	0.008	-	-	7,670
0.211	H.T.S.	29.2	0.008	-	-	7,780

Table 3. Summary of Material Properties of Bracing Rods.

```
C              PROGRAM TRUEFORCE
C      THIS PROGRAM COMPUTES THE TRUE FORCE
C      IN A PLASTICALLY DEFORMING BRACE WHEN
C      A DEFINED TRUE DEFLECTION IS IMPOSED
C
       REAL BLC,ABO,DL,AMDL,FN,FKO,TRFOR
    6  READ(5,1) BLC,ABO,DL,AMDL,FN,FKO
    1  FORMAT(6G10.0)
       WRITE(6,2) BLC,ABO,DL,AMDL,FN,FKO
    2  FORMAT(25X,'INPUT DATA ECHO CHECK',6G10.3///)
       ADL=DL
    5  BLI=BLC+ADL
       ABI=ABO*BLC/BLI
       TRFOR=ABI*FKO*((ALOG(BLI/BLC))**FN)
       WRITE(6,3) ADL
    3  FORMAT(25X,'TRUE DEFLECTION  =  ',E12.4,'INCHES'//)
       WRITE(6,4) TRFOR
    4  FORMAT(25X,'TRUE FORCE       =  ',E12.4,'LBS.'////)
       ADL=ADL+DL
       IF((AMDL-ADL).GE.0) GO TO 5
       GO TO 6
       END
```

```
INPUT DATA ECHO CHECK  45.0     0.490E-01 0.782E-01  2.00     0.250     0.154E 06
```

TYPICAL OUTPUT

```
TRUE DEFLECTION  =    0.782OE-01INCHES

TRUE FORCE       =    0.1537E 04LBS.


TRUE DEFLECTION  =    0.1564E 00INCHES

TRUE FORCE       =    0.1825E 04LBS.


TRUE DEFLECTION  =    0.2346E 00INCHES

TRUE FORCE       =    0.2016E 04LBS.


TRUE DEFLECTION  =    0.3128E 00INCHES

TRUE FORCE       =    0.2162E 04LBS.
```

LIST 2

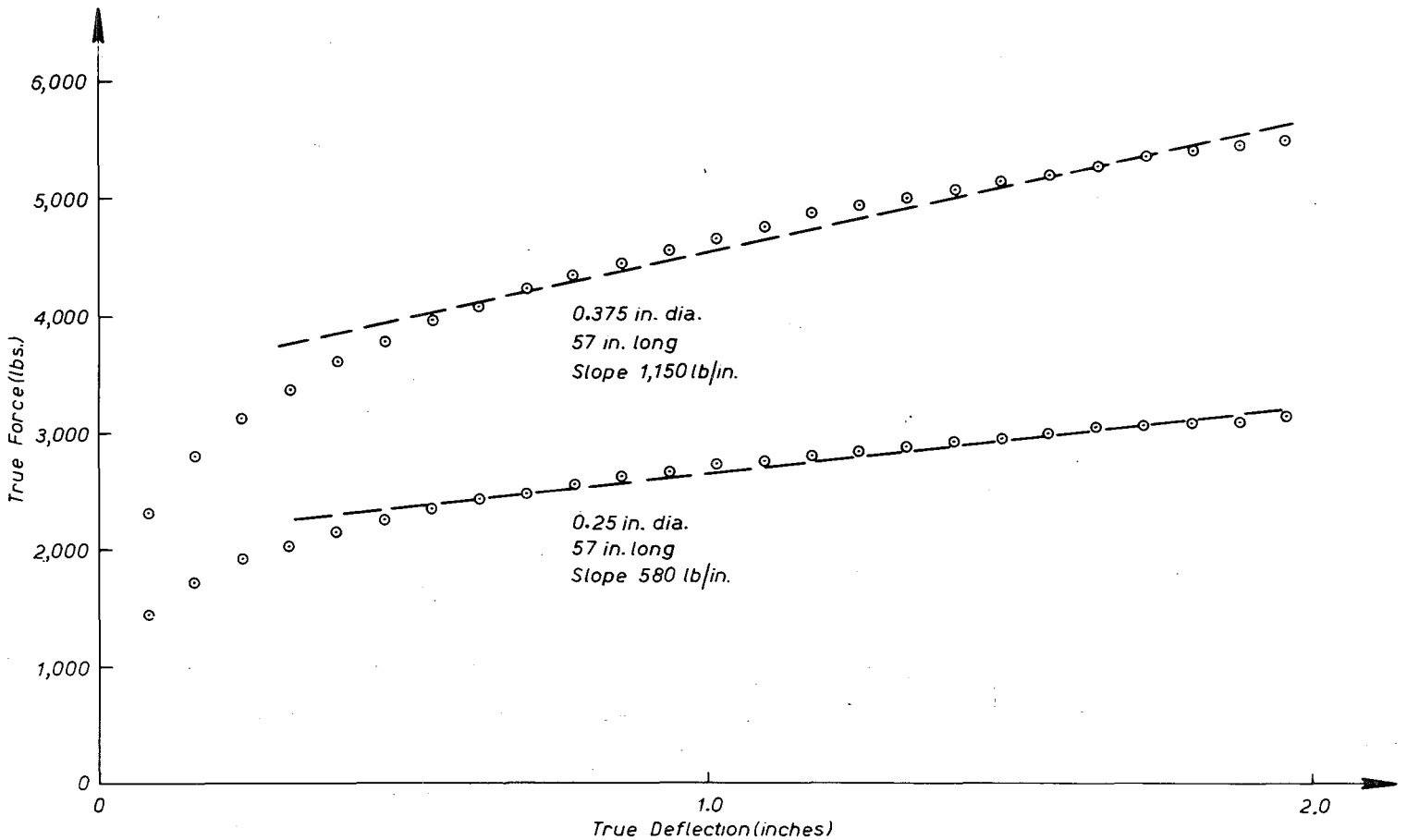

Fig. 3.6: TRUE FORCE / TRUE DEFLECTION CURVES FOR MILD STEEL BRACING RODS AFTER YIELD

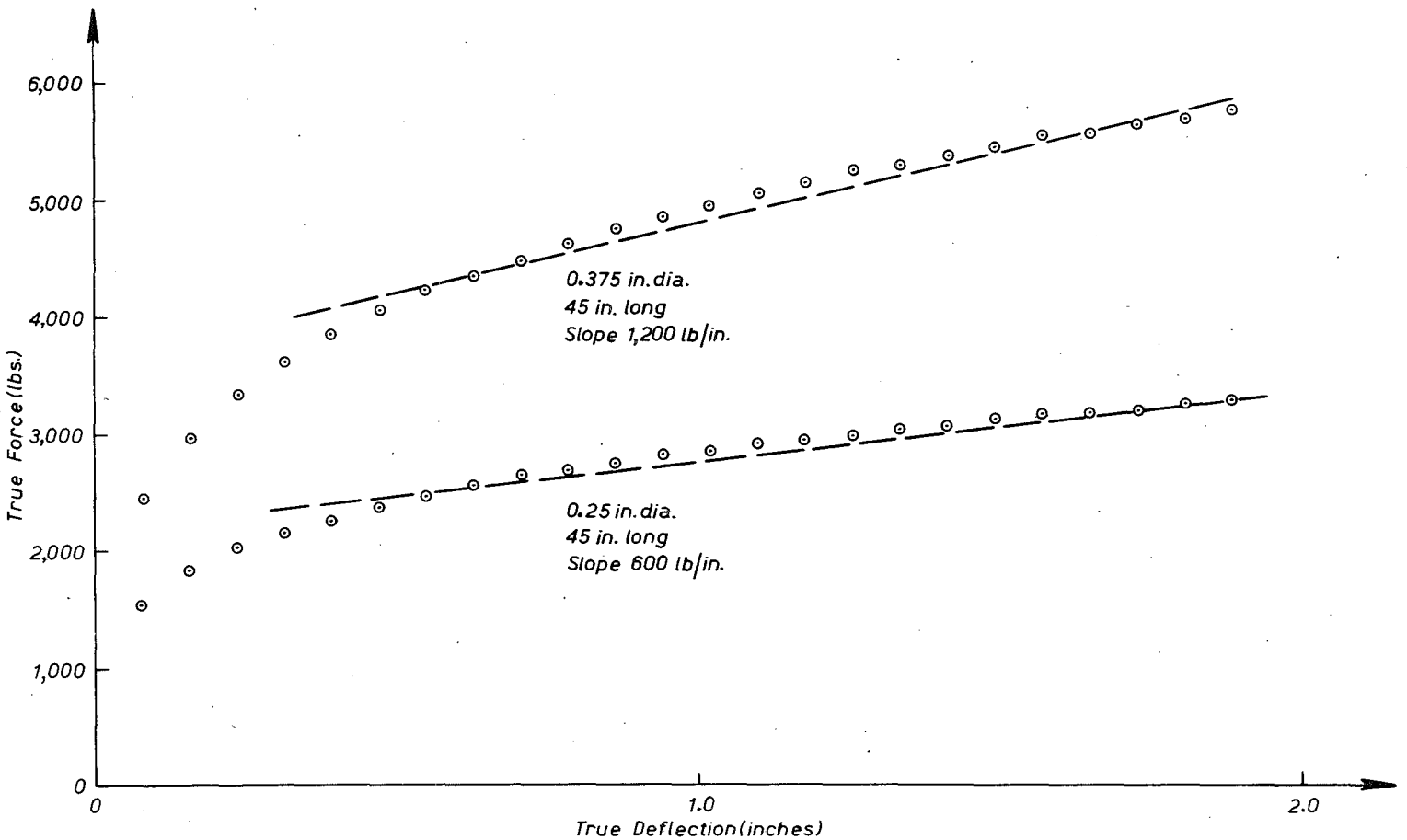

Fig. 3.7 : TRUE FORCE / TRUE DEFLECTION CURVES FOR MILD STEEL BRACING RODS AFTER YIELD

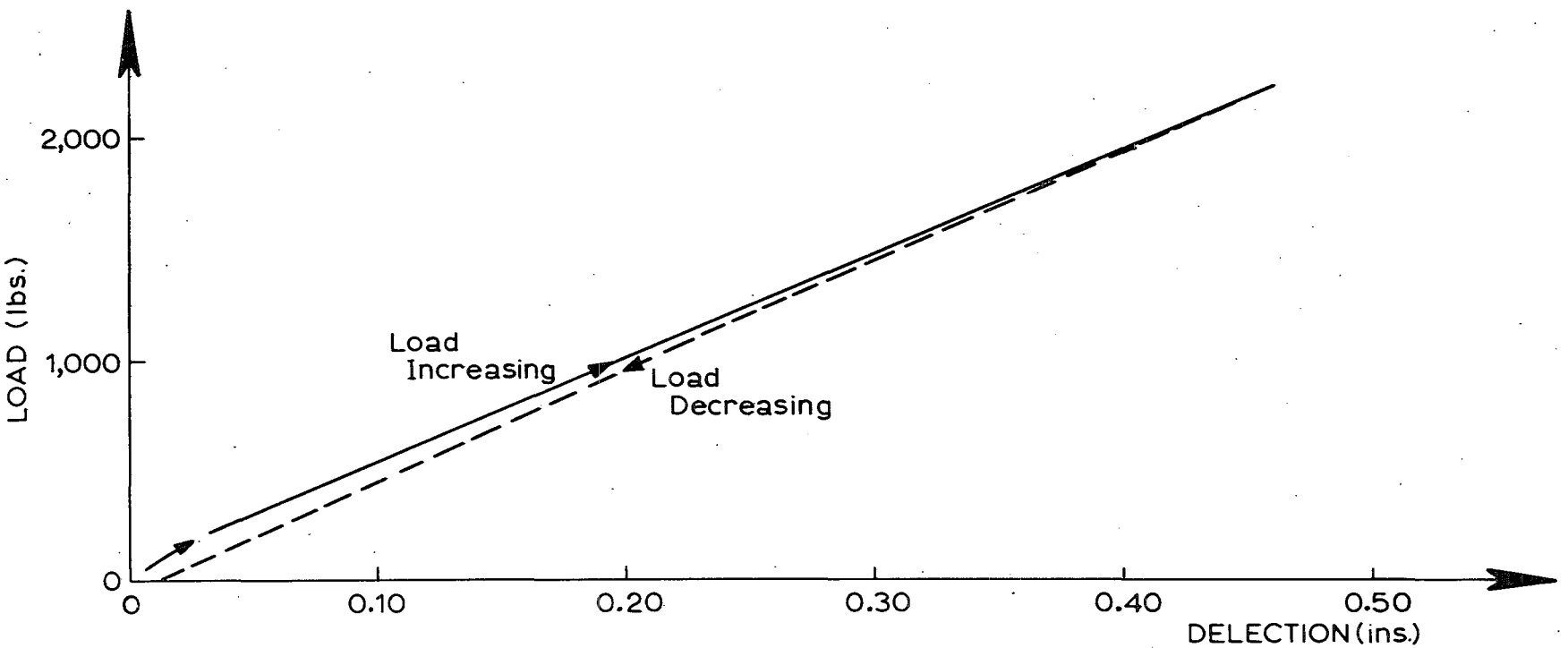

Fig. 3.8: STATIC SIDESWAY STIFFNESS TEST – UNBRACED RIGID KNEE PORTAL

load/deflection graph shown in Figure 3.9 to be obtained. The initial experimental stiffness of 20,000 lb/in. compares reasonably well with the theoretical value (4,570 lb/in. for the bare frame plus 16,350 lb/in. for the bracing giving a total of 20,920 lb/in.) and the slope of the second section of the graph, 5,000 lb/in., correlates well with the predicted value (4,570 lb/in. for the bare frame plus 580 lb/in. for the post yield axial stiffness of the bracing, see Figure 3.6, giving a total of 5070 lb/in.).

As had been anticipated, considerable difficulty was experienced at the start of the test on the braced frame in adjusting the bracing to take up the slack in the anchorages and to be "just tight" at the commencement of the racking test. After many attempts to overcome this problem a satisfactory technique was devised. This involved repeated tensioning and releasing of the bracing by means of the adjustment screw at the lower end, until readings across the Demac gauge points indicated that no further slip in the anchorage was occurring (confirmed by a linear increase in strain reading resulting from equal incremental rotations of the adjustment screw) and finally the screw was adjusted to the position at which strain readings taken across the Demac points indicated that the "just tight" condition existed.

A subsequent static racking test on a pinned knee frame braced with one 0.25 in. diameter mild steel rod indicated that the elastic lateral stiffness was 16,700 lb/in. When the same frame was braced with two diagonally opposed 0.25 in. diameter mild steel rods, with equal residual prestress achieved through use of the adjustment screws

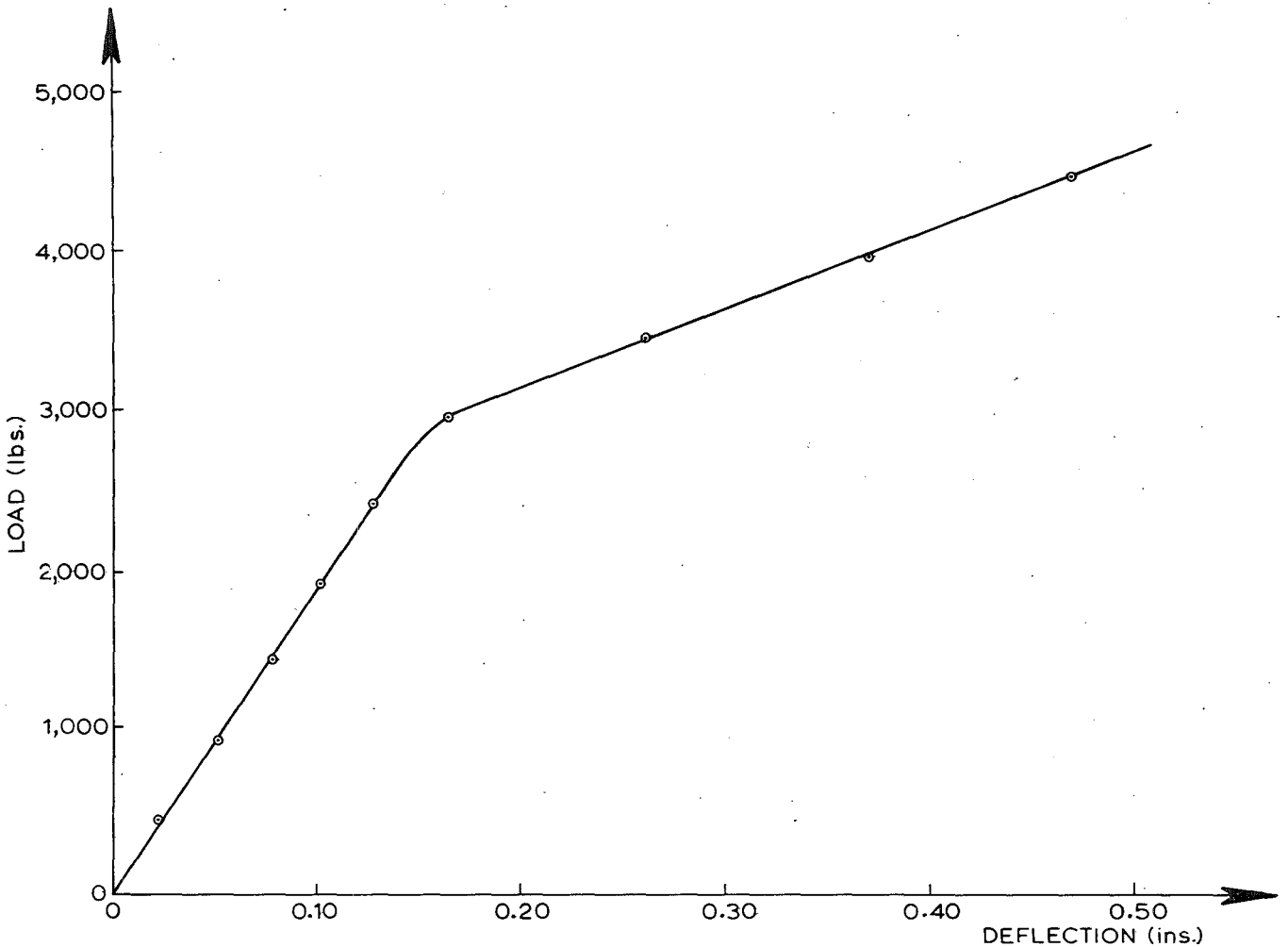

Fig. 3.9 : STATIC SIDESWAY STIFFNESS TEST RIGID KNEE PORTAL BRACED WITH ONE 0.25 in. dia. M.S. ROD.

after the initial slack in the anchorages had been removed, the load/deflection curve shown in Figure 3.10 was determined experimentally. This confirmed the expected behaviour and, in particular, illustrated the increased stiffness conferred by the prestressed member, the return to the characteristics of a frame with a single brace when the second rod became just slack on losing its prestress and the onset of yield in the tensioned rod at a value of racking force consistent with the incremental load due to the racking being superimposed on the initial prestress in the rod.

A static racking test on a pinned knee frame braced with two diagonally opposed 0.205 in. diameter high tensile steel rods as well as two mild steel rods, all four rods being initially prestressed, enabled the load/deflection curve shown in Figure 3.11 to be obtained. The initial stiffness of 55,000 lb/in. correlates well with the theoretical value (33,000 lb/in. plus 10,300 lb/in. for each of the two H.T.S. rods), the stiffness of 10,700 lb/in. corresponds to the situation which only one (yielding) mild steel rod (of computed stiffness 580 lb/in.) and one high tensile steel rod (computed stiffness 10,300 lb/in.) are together resisting the sidesway, and the 600 lb/in. stiffness corresponds to the situation in which only one yielding mild steel rod remains active. Confirmation that two of the rods went slack soon after point I on the curve, that the tensioned mild steel rod commenced yielding just above point I and had reached a fully yielding state and point II and that the tensioned high tension steel rod fractured at point III was

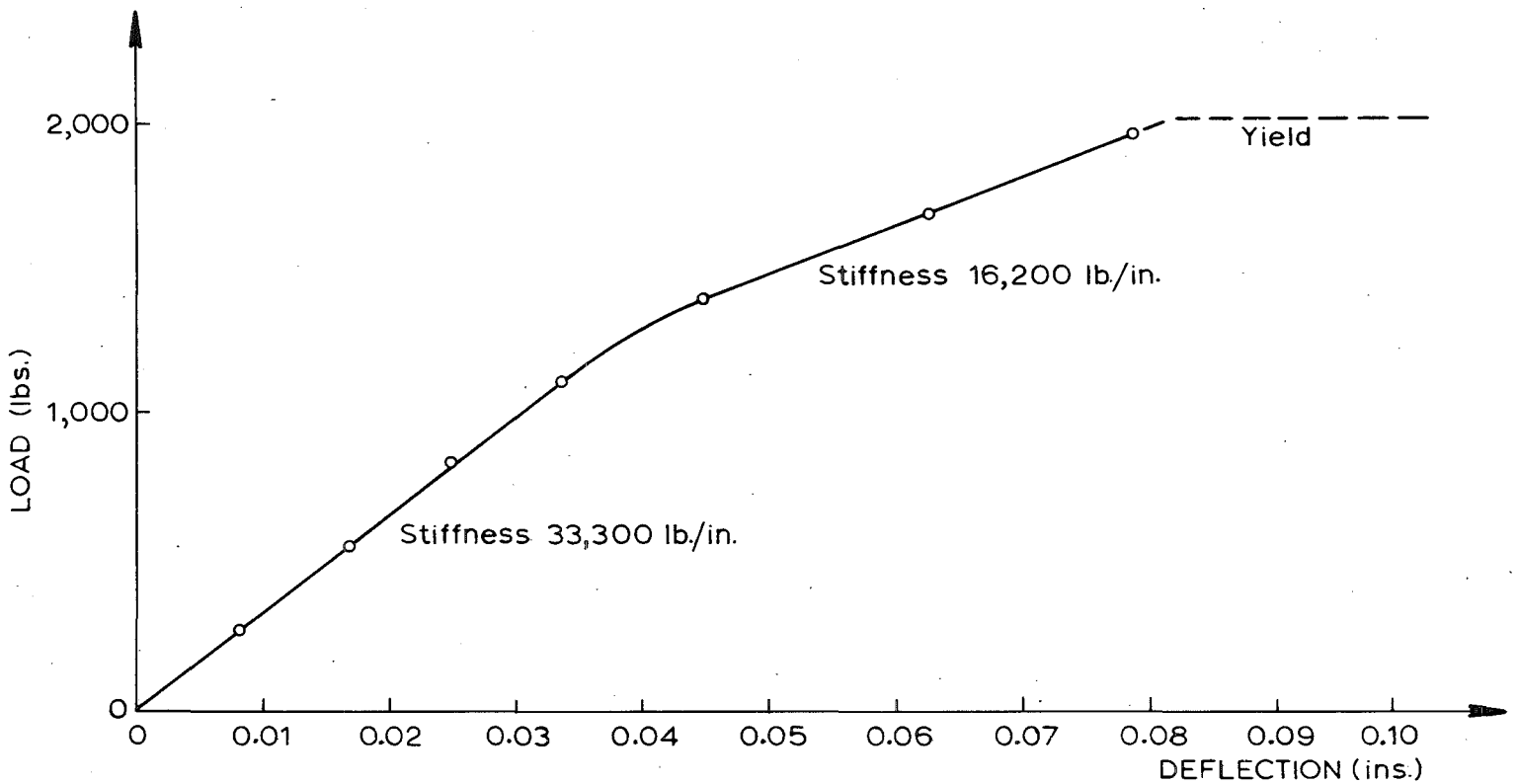

Fig. 3.10: STATIC SIDESWAY STIFFNESS TEST PINNED KNEE PORTAL BRACED WITH DIAGONALLY OPPOSED PRESTRESSED 0.25 dia. M.S. RODS.

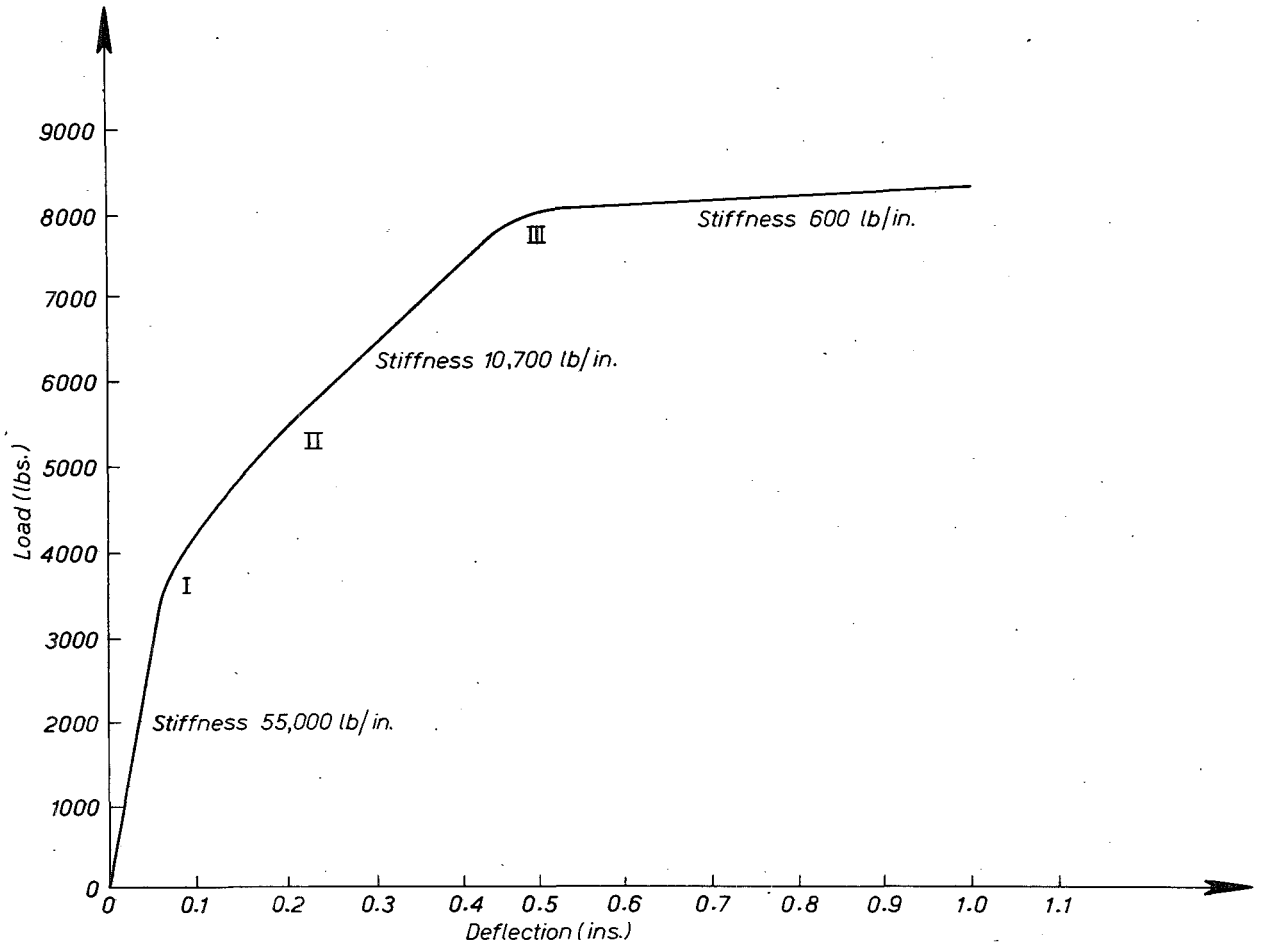

Fig. 3.11: STATIC SIDESWAY STIFFNESS TEST. PINNED KNEE PORTAL BRACED WITH DIAGONALLY OPPOSED PRESTRESSED 0.25 in. dia. M.S. RODS AND PARALLEL DIAGONALLY OPPOSED 0.205 in. dia. H.T.S. RODS.

provided by the Demac gauge readings which were undertaken throughout the test. These readings were found to be almost indispensable in interpreting the sequence of behaviour in this and similar tests.

Several static racking tests involving reversed (cyclic) loading were subsequently undertaken on the pinned knee frame braced with four prestressed rods. Typical behaviour patterns are shown in Figure 3.12 and 3.13. In Figure 3.12, 3.5 cycles of loading were imposed. In cycle A the system retained its initial characteristics throughout. No rods completely lost their prestress nor did any yield; the constant stiffness of 55,000 lb/in. determined experimentally correlating satisfactorily with the predicted value. In cycle B the initial prestress in the mild steel rod was eliminated at a sway of 0.05 in. and consequently this rod became slack; the high tensile steel rod followed this behaviour pattern at almost the same value of sway and consequently the slope of the upper section of the cycle B curve (27,000 lb/in.) represents the situation in which the pinned knee frame is braced with the two tensioned rods acting alone. On removal and subsequent reversal of load the cycle B curve returns along lines of similar gradient, corresponding to the participation of either two or four bracing rods, producing an essentially bilinear trace. Since no yielding of the rods was evident it is concluded that the failure of the downward trace to superimpose on the upward one is primarily due to the inevitable slack in the pinned frame. This feature shows up again at the bottom of cycle B curve.

Cycles C and D exhibited essentially trilinear characteristics.

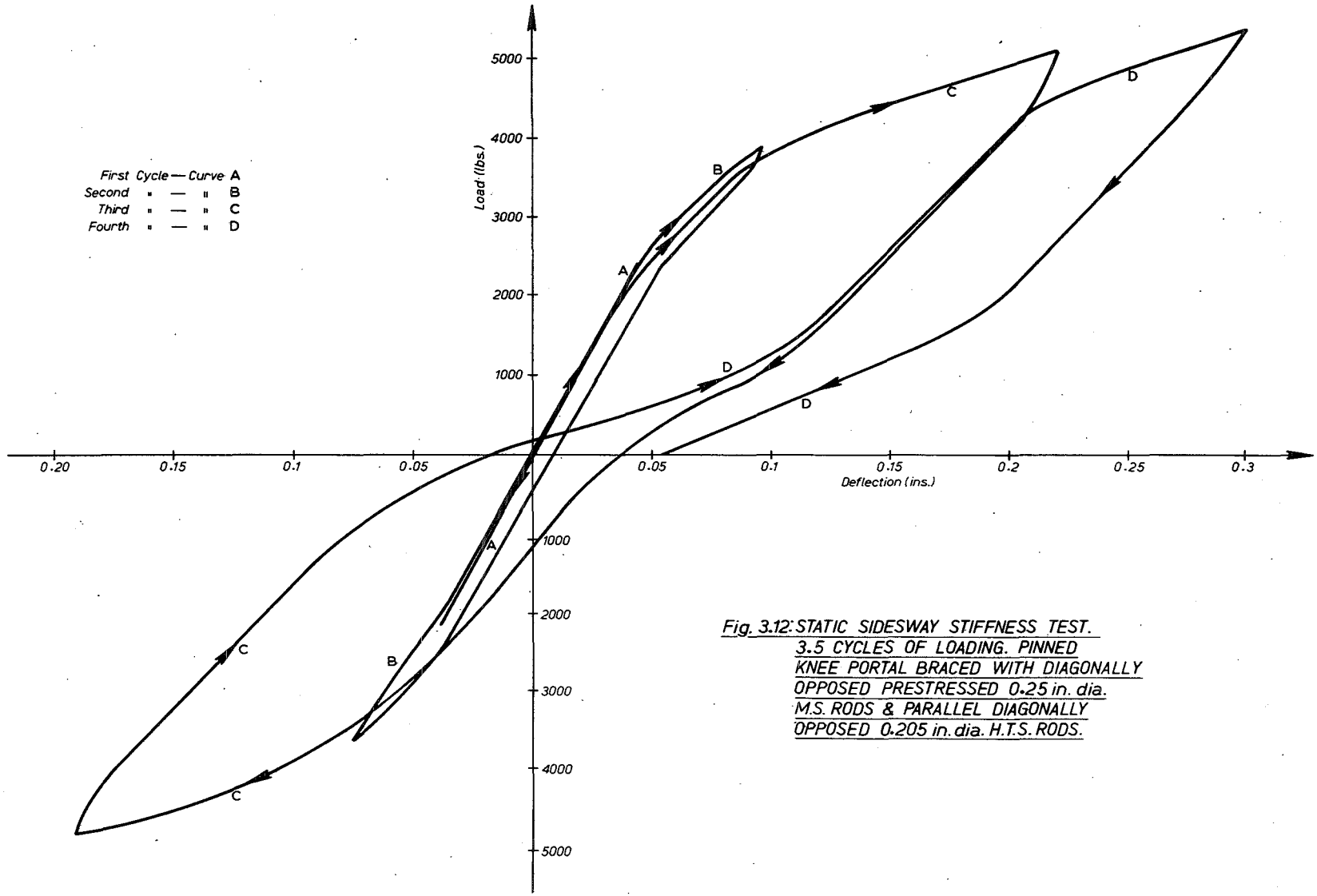

Fig. 3.12: STATIC SIDESWAY STIFFNESS TEST. 3.5 CYCLES OF LOADING. PINNED KNEE PORTAL BRACED WITH DIAGONALLY OPPOSED PRESTRESSED 0.25 in. dia. M.S. RODS & PARALLEL DIAGONALLY OPPOSED 0.205 in. dia. H.T.S. RODS.

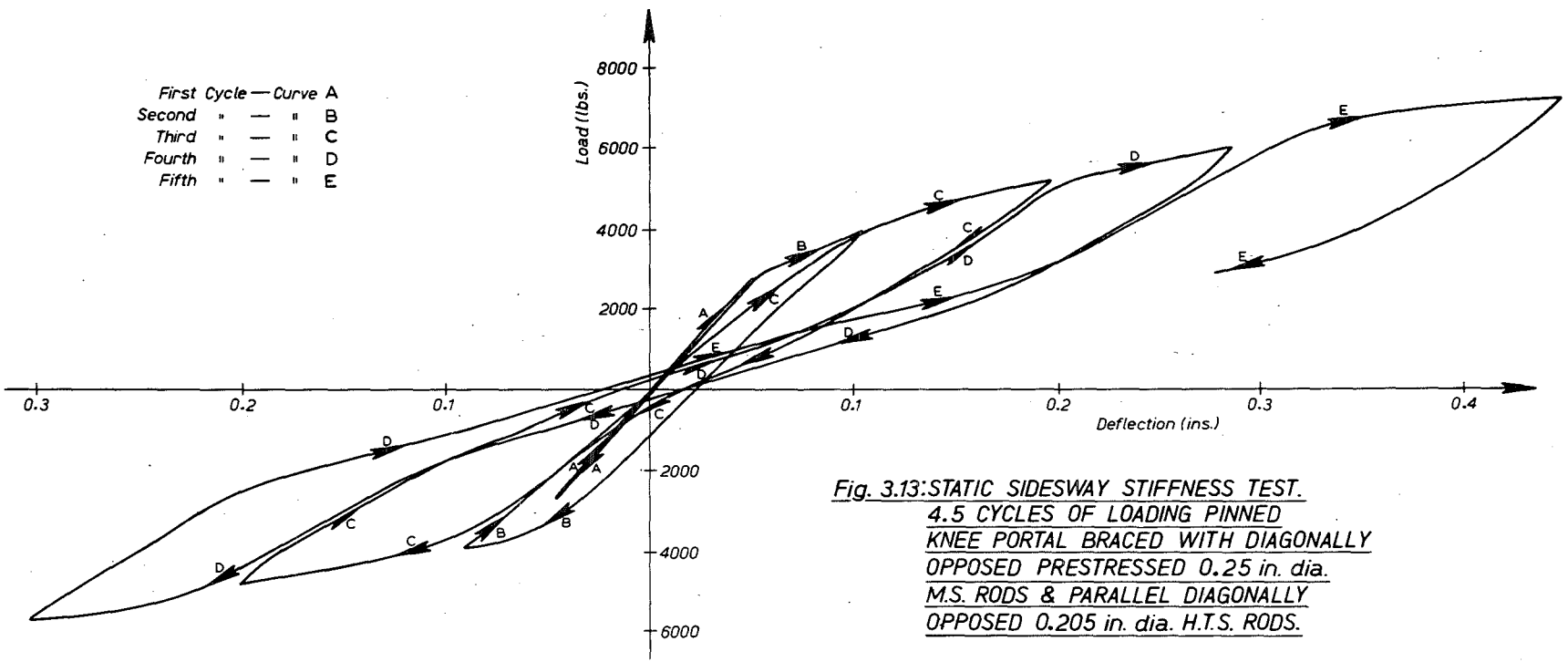

Fig. 3.13: STATIC SIDESWAY STIFFNESS TEST. 4.5 CYCLES OF LOADING PINNED KNEE PORTAL BRACED WITH DIAGONALLY OPPOSED PRESTRESSED 0.25 in. dia. M.S. RODS & PARALLEL DIAGONALLY OPPOSED 0.205 in. dia. H.T.S. RODS.

The slopes corresponding to 55,000 lb/in. and 27,000 lb/in. stiffnesses reflecting the participation of four and two braces respectively, acting elastically, and the slopes of 10,500 lb/in. results from the yielding of the tensioned mild steel braces leaving one high tensile steel rod to provide the majority of the sway resistance (combined computed stiffness of 10,880 lb/in.). The width of the C and D cycle curves can be accounted for primarily by the yielding of the tensioned mild steel bracing rods; this action provides the major part of the energy absorbing property of the system.

After replacing the bracing members and reintroducing prestress into them by following the setting up procedure described earlier, 4.5 cycles of loading were applied to the pinned knee frame with the result shown in Figure 3.13. Characteristic slope values of 55,000 lb/in., 10,500 lb/in. and 27,000 lb/in. were again established and, as in the 3.5 cycle test, in each case these may be satisfactorily related to the state of participation and of stress of the bracing members.

Static racking tests involving reversed (cyclic) loading or braced rigid knee frames were also carried out. Typical results are shown in Figures 3.14(a) and 3.14(b). A rigid knee frame was prestressed with two pairs of diagonally opposed rods, as in the earlier tests on the pinned frame, and the results of the first two cycles of loading are presented in Figure 3.14(a). The slope of the first - entirely elastic - cycle of 57,500 lb/in. correlates satisfactorily with the predicted value (53,600 lb/in. for the bracing plus 4,570 lb/in. for the bare frame) and the characteristic slopes of 12,000 lb/in. and

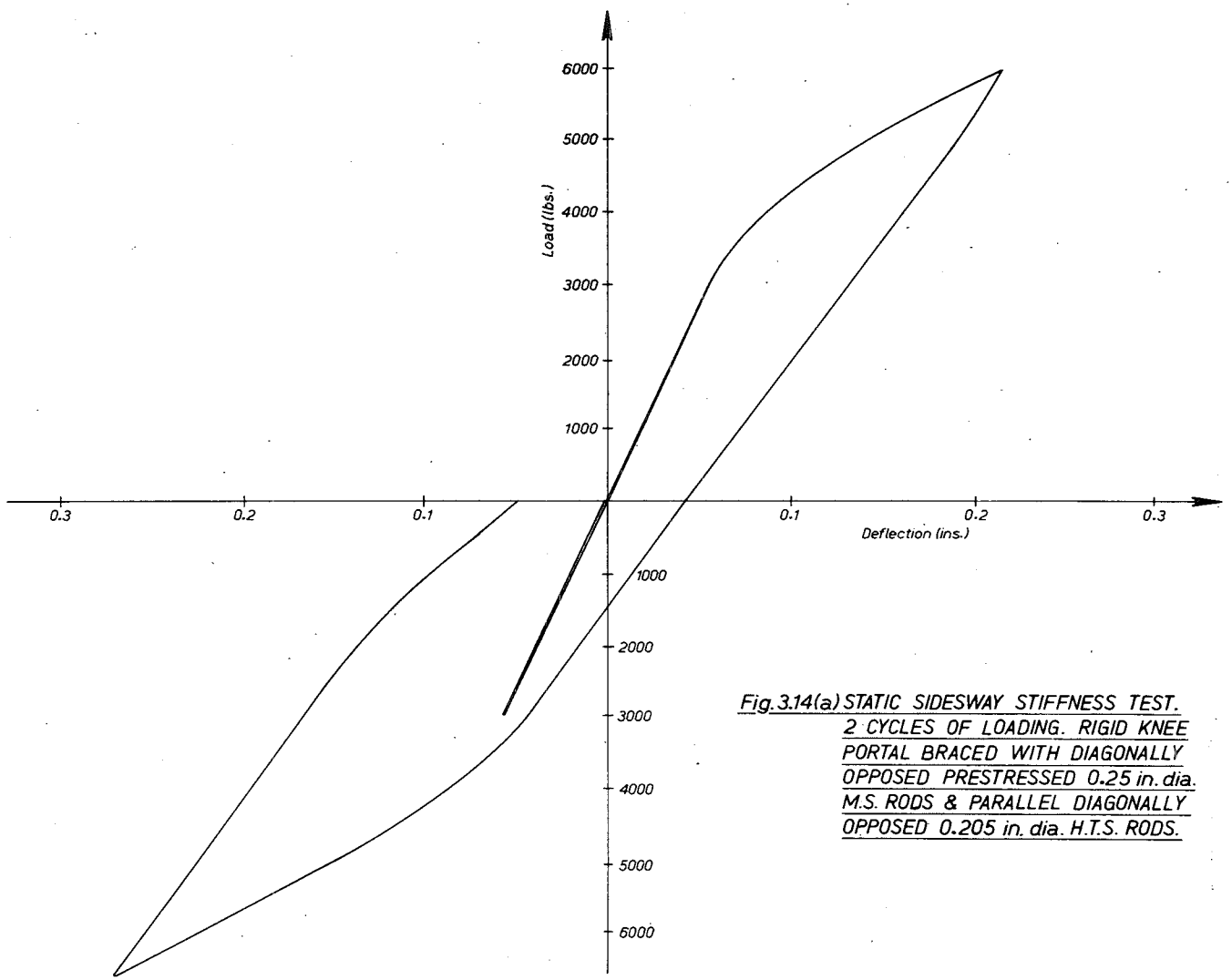

Fig. 3.14(a) STATIC SIDESWAY STIFFNESS TEST. 2 CYCLES OF LOADING. RIGID KNEE PORTAL BRACED WITH DIAGONALLY OPPOSED PRESTRESSED 0.25 in. dia. M.S. RODS & PARALLEL DIAGONALLY OPPOSED 0.205 in. dia. H.T.S. RODS.

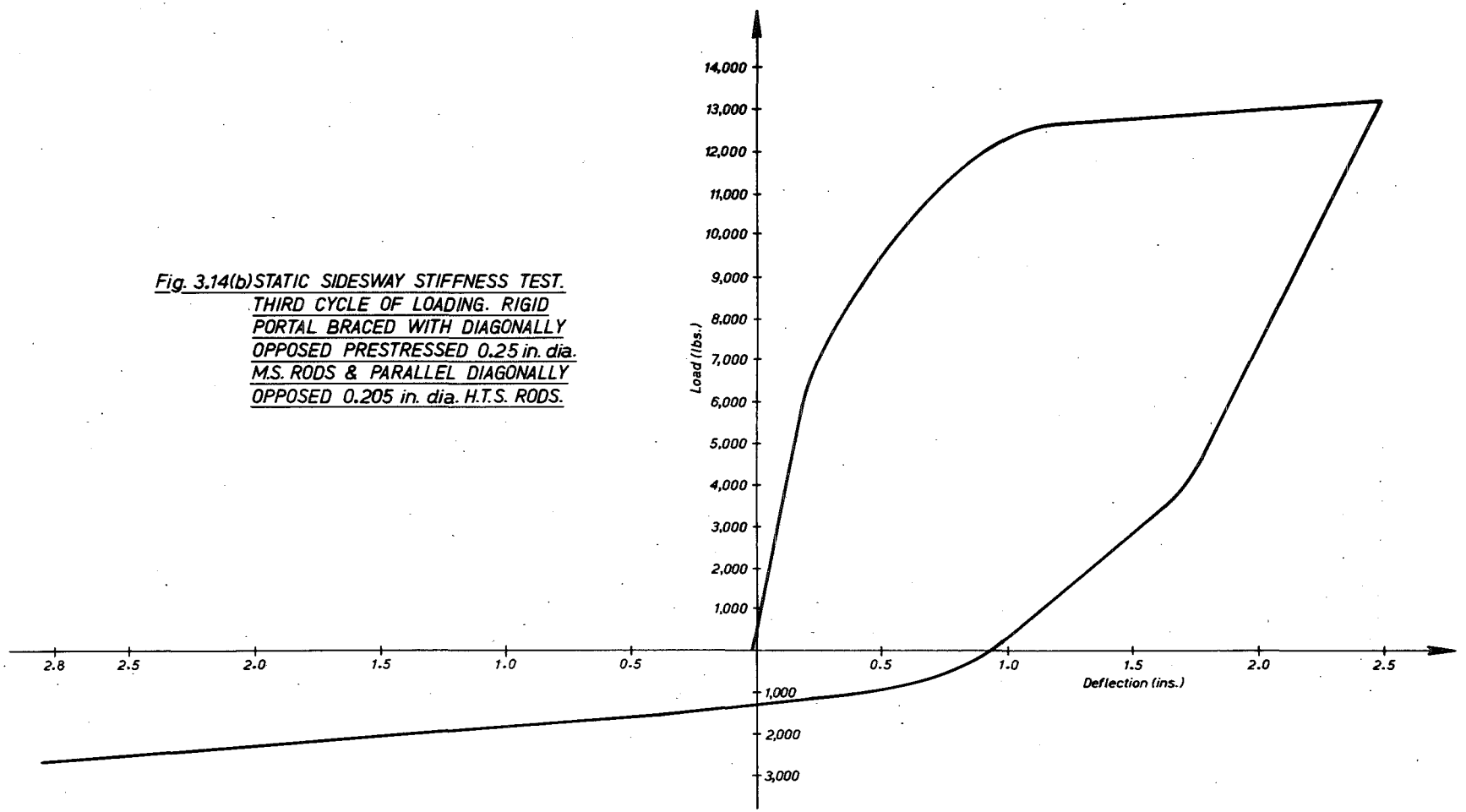

Fig. 3.14(b) STATIC SIDESWAY STIFFNESS TEST. THIRD CYCLE OF LOADING. RIGID PORTAL BRACED WITH DIAGONALLY OPPOSED PRESTRESSED 0.25 in. dia. M.S. RODS & PARALLEL DIAGONALLY OPPOSED 0.205 in. dia. H.T.S. RODS.

34,000 lb/in. exhibited by the second cycle of loading correspond to the situations in which the sidesway is resisted in the first case by a yielding frame and a yielding mild steel rod (580 lb/in.) acting in parallel with one high tensile steel rod (10,300 lb/in.) and in the second case by one mild steel rod and one high tensile steel rod together with the frame, all acting elastically (16,300 lb/in. plus 10,300 lb/in. plus 4,570 lb/in.). Subsequent cyclic loading of this frame resulted in the curve presented in Figure 3.14(b) being obtained. In this case the very large deflections both made deflection measurement difficult and resulted in almost continuously varying inelastic characteristics being determined, as distinct from phased inelastic behaviour. The essentially linear portions of the curve having slopes of 21,000 lb/in., 12,000 lb/in. and 300 lb/in. are difficult to relate satisfactorily to the calculated element stiffness values although the 12,000 lb/in. stiffness corresponds to the sidesway being resisted by one high tensile rod assisted by a yielding frame and a yielding mild steel rod. One high tensile rod fractured at the top of the loading cycle and the effect of this was to jar the whole rig and to necessitate the resetting of the dial gauges. Consequently some doubt exists in the relationships of the later readings to the earlier ones.

Subsequent to the static reversed (cyclic) loading experiments, a series of braced frames were tested using the dynamic loading machine. Since this is basically a controlled displacement device, as distinct from a controlled load testing machine, it proved to be of only limited use in complementing the experiments undertaken

statically. Nevertheless its availability prompted a series of tests on both pinned and rigid knee frames braced with diagonally opposed pairs of rods. The machine was run at a constant speed throughout, to provide a connecting rod frequency of 2.2 c.p.s. which was selected to lie within the normally accepted range of earthquake induced vibration and the frames were subjected to cyclic displacements through a range of selected amplitudes. The normal test pattern involved increasing, and in some cases subsequently decreasing, the amplitudes by equal steps and recording the sidesway load/deflection characteristics on the X-Y plotter.

Several typical recorded curves are presented in Figure 3.15. Curve (i) shows a single cycle of a test of a pinned knee portal braced with one 0.25 in. diameter mild steel rod in each direction. As a result of previous loading and consequent yielding of the bracing about 0.3 in. of slack exists on each side of the initial undeflected position. Hence the rods only contribute stiffness in the extremities of the oscillation when an experimental value of stiffness corresponding satisfactorily to the calculated one was observed. Curves (ii) to (vii) in Figure 3.15 illustrate the effect of increasing the amplitude of movement applied by the test machine at a constant rate. The gradient of the top of each loop corresponds to the stiffness of the yielding mild steel brace and the essentially bilinear nature of the response of this system is once again apparent. In curve (viii) in Figure 3.15 the load/deflection loop corresponding to the last cycle in the overlaying group of curves is recorded.

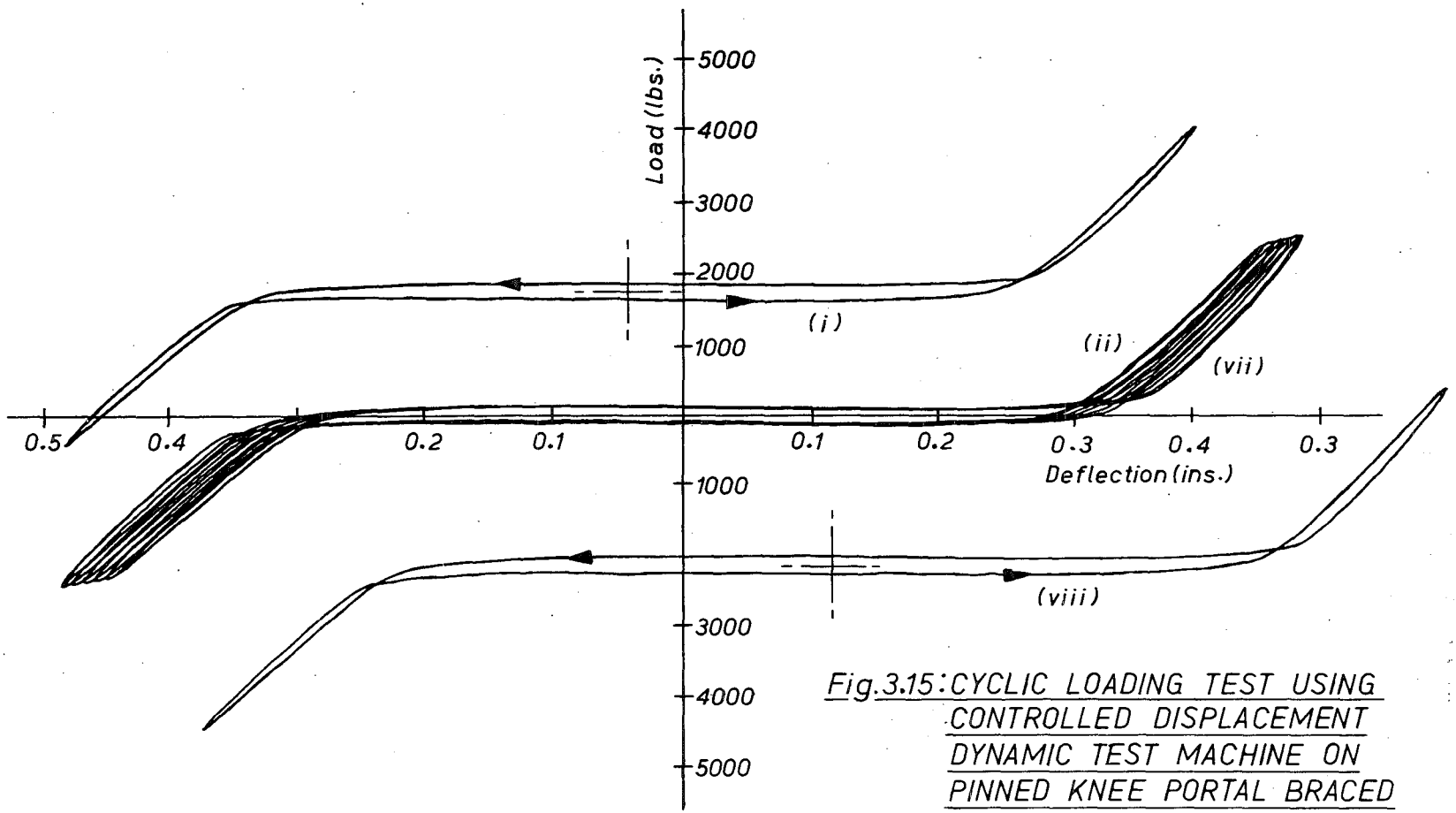

Fig.3.15: CYCLIC LOADING TEST USING CONTROLLED DISPLACEMENT DYNAMIC TEST MACHINE ON PINNED KNEE PORTAL BRACED WITH TWO DIAGONALLY OPPOSED 0.25 in. dia. M.S. RODS.

The result of four cycles of loading of a pinned knee frame, braced with one pair of 0.25 in. diameter mild steel rods and one pair of 0.211 in. diameter high tensile steel rods, is shown in Figure 3.16. The loading machine was adjusted at a constant rate to give four loops of decreasing amplitude subsequent to the braced frame having been vibrated through displacements of considerably larger amplitudes. The characteristics of a stiffening system are apparent from the curves; the first gradient on the rising section of each loop corresponds to the situation in which one 0.211 in. diameter high tensile steel brace acting alone supplies the sidesway stiffness (computed at 10,900 lb/in.) and the steeper slope is appropriate to the configuration in which one mild steel brace is also acting (the slack in the mild steel braces arising from previous yielding having been taken up in the first portion of the sway movement).

The curve in Figure 3.17 was recorded during a cyclic loading test on a rigid knee frame braced with both mild steel and high tensile steel rods. A stiffening characteristic is once more evident. Again this is due to the participation at the extremities of the oscillatory movement of the previously yielded mild steel rods. The gradient across the major portion of the curve (15,800 lb/in.) correlates with the computed value of 10,900 lb/in. (for the high tensile steel rod) plus 4,570 lb/in. (for the bare frame). The steeply rising section of the loading cycle (slope 32,000 lb/in.) compares satisfactorily with the predicted value made up of 10,900 lb/in. and 4,570 lb/in. plus 16,700 lb/in. for the

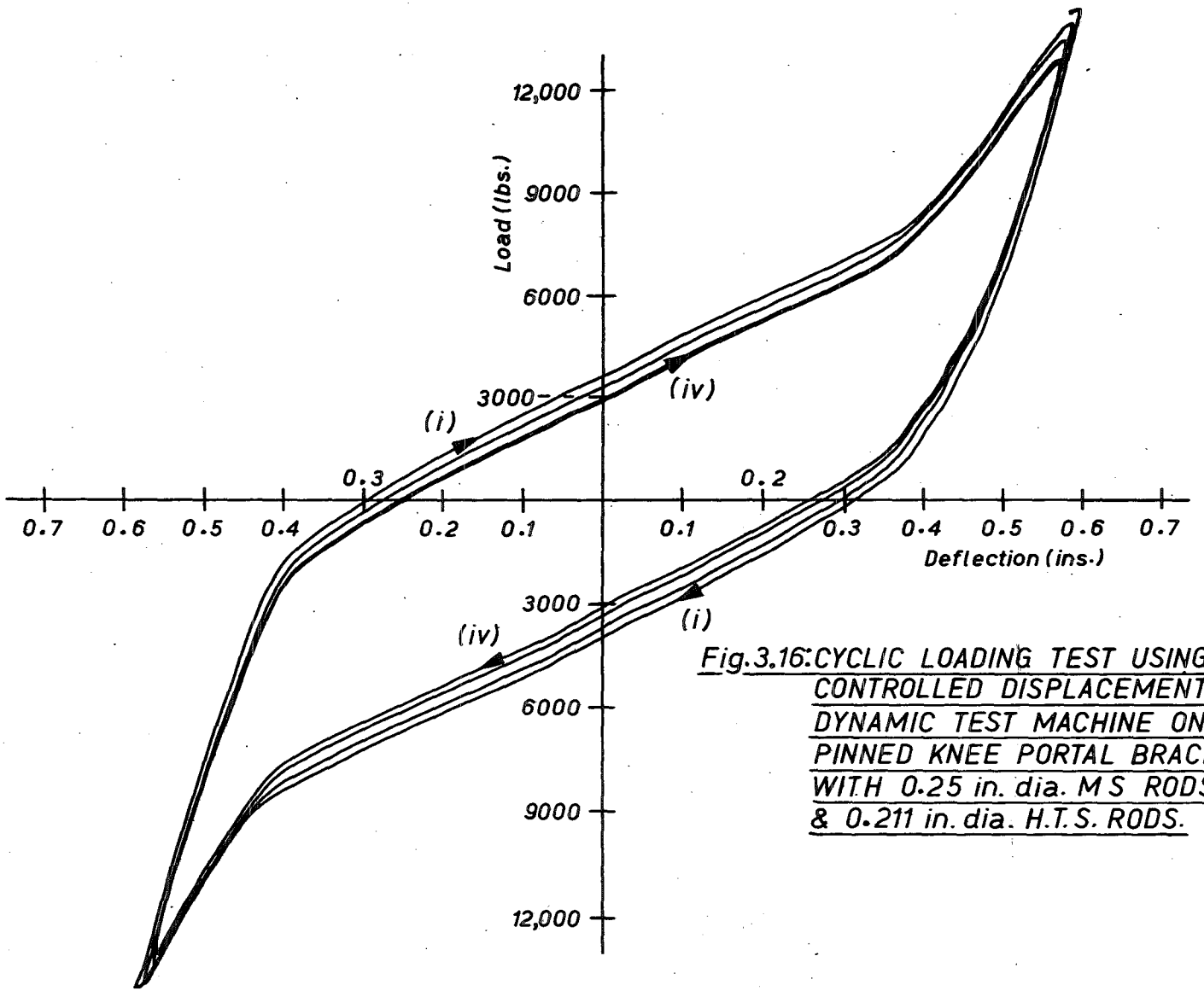

Fig. 3.16: CYCLIC LOADING TEST USING CONTROLLED DISPLACEMENT DYNAMIC TEST MACHINE ON PINNED KNEE PORTAL BRACED WITH 0.25 in. dia. M S RODS & 0.211 in. dia. H.T.S. RODS.

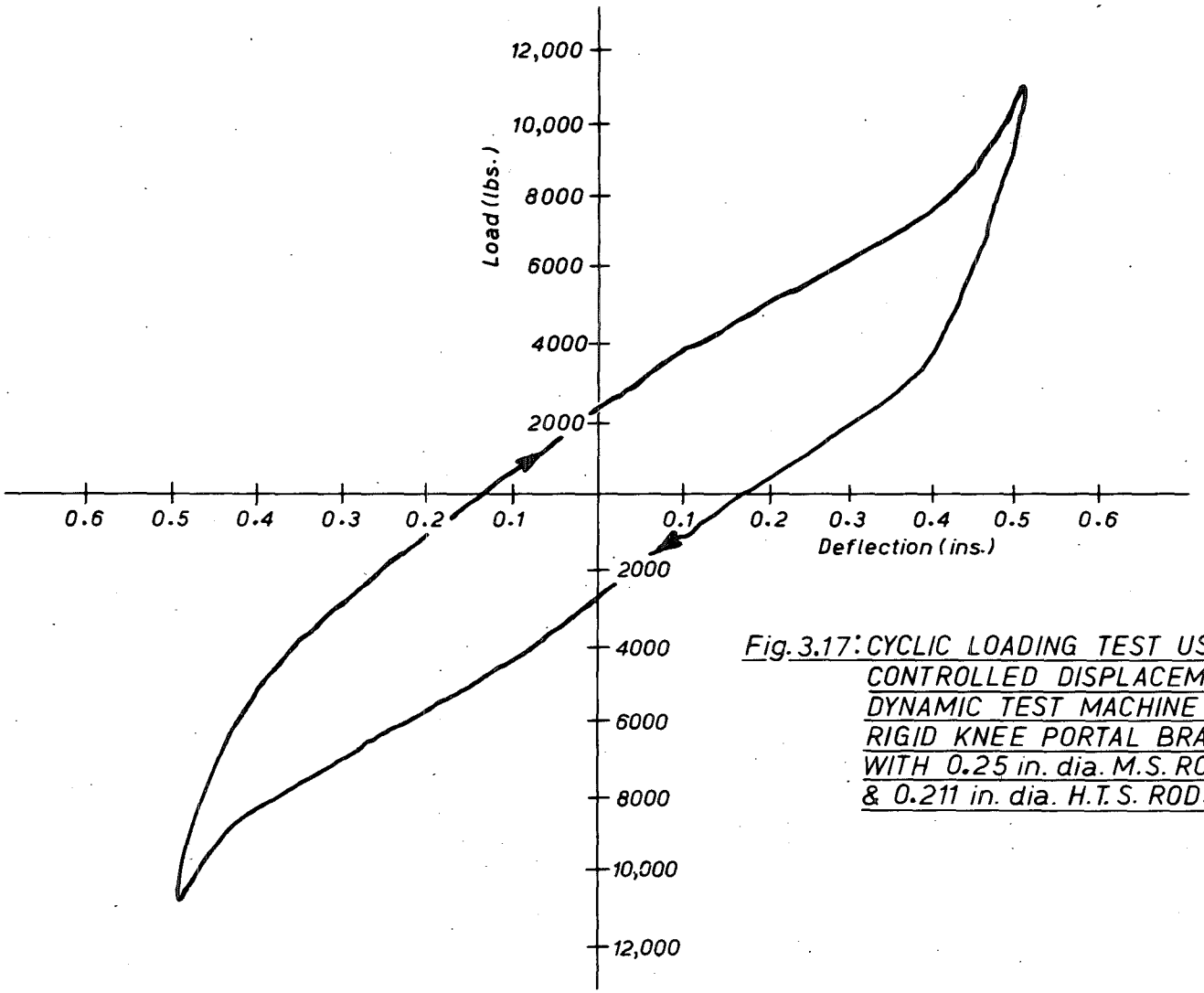

Fig. 3.17: CYCLIC LOADING TEST USING CONTROLLED DISPLACEMENT DYNAMIC TEST MACHINE ON RIGID KNEE PORTAL BRACED WITH 0.25 in. dia. M.S. RODS & 0.211 in. dia. H.T.S. RODS.

tensioned mild steel brace.

III.5 Comment on Cross Braced Panel Testing

Points established as a result of the testing included the fact that, once a satisfactory technique had been developed, no real problem was encountered in ensuring that the bracing anchorages were secure or in introducing a chosen level of prestress into the braces. Another point worthy of recording was that in the testing undertaken in the static rig the readings made on the bracing rod Demac gauges proved to be of great assistance when the load/deflection curves were being interpreted. These readings were notably more reliable than those made using the wire resistance strain gauges, probably because of the problems arising from local flexing and yielding of the rods under the resistance strain gauges.

The tests established that selected multiphase response may be achieved reliably in cross braced frames by the choice of suitable bracing and that, in particular, a significant variation in sidesway load/deflection characteristics may be obtained by the use of sacrificial members, possibly combined with initial prestress in some or all of the braces. It has been shown to be a practical possibility to produce a braced frame which is relatively stiff under small sidesway, which becomes more flexible at larger deflections and concurrently exhibits ductile properties, but which has a reserve of energy absorption capacity coupled with the ability to stiffen up again at near destructive levels of sidesway.

CHAPTER FOUR

SEISMIC ANALYSES BY DIGITAL COMPUTER MATHEMATICAL MODELLING

IV.1 Introduction

The increasing availability of electronic digital computers throughout the last twenty years has prompted the solution of many previously intractable problems, one of which is the analysis of the response of structures to time varying ground disturbances resulting from earthquakes. Early attempts to apply digital computers to structural engineering problems almost invariably involved programming the previously well tried manual methods of analysis, whereas much more efficient computer orientated techniques were subsequently devised. To some extent this pattern was reflected in the particular application to seismic response calculations. The response spectra, normal mode approach[47] was developed a generation ago in an attempt to simplify earthquake response analyses[48,49,50,51] and initial applications of digital computers in earthquake engineering involved their use to determine response spectra[52] and modal properties[6]. However digital computers have made the direct numerical integration of the equations of motion of multidegree of freedom system a practical proposition. This fact has prompted the use of a numerical integration approach to seismic analysis calculations, particularly to inelastic response calculations[53,54] where the system is assumed to behave in a linear elastic manner during each

time increment used in the stepwise numerical integration process and the non linear behaviour is determined as the response of a sequence of successively differing systems.

IV.2 Mathematical Modelling of Structural Frameworks

The steps involved in mathematical modelling of a structure and the solution of the distribution of member forces and distortions resulting from a chosen applied load may be summarised as:-

(1) Idealisation of the structure to a system readily represented mathematically

(2) Evaluation of the structural properties of the elements

(3) Combination of the element characteristics to determine the properties of the complete system

(4) Solution of the equations describing the load/distortion relationships of the complete system so determining the deformations (or forces) resulting from the applied loading

(5) Calculation of the effects on individual members of the applied loading.

In the case of plain frameworks step (1) consists of idealising the structure to a series of one dimensional elements, the properties of which are assumed to be concentrated along the centre-line axis. These elements are considered to be connected together at the joints or nodal points where the mass of the structure is assumed to be concentrated and where inter-element continuity is enforced.

Step (2) involves the determination of stiffness (or flexibility)

coefficients making use of the elastic properties of the element material and the geometric characteristics (length, area, orientation). Typically Slope/Deflection relationships are invoked to determine element stiffness matrices [k] which consist of arrays of element stiffness values each being the force required to produce unit deformation for a particular degree of freedom[55,56]. These element stiffness characteristics may subsequently be combined to determine the stiffness of the complete framework by summing the stiffnesses of the ends of all members meeting at a joint (and making allowance for the carry-over effect to adjacent joints). In matrix algebra terms step (3) results in the calculation of a frame stiffness matrix [K] consisting of an array of stiffnesses each relating the force acting in the sense of a particular degree of freedom to the corresponding unit deformation.

The equation relating the applied loading system {L} to the distortion of the whole structure {d} namely

$$\{L\} = [K] \cdot \{d\}$$

may be solved in step (4) by inversion, iteration or relaxation techniques[57]. Once the distortion of the whole structure under the chosen loading has been established, in terms of deflections and rotations in the senses of the degrees of freedom originally selected, back substitution in the element stiffness relationships enables the individual member forces to be determined.

In matrix terms this step (5) may be expressed

$$\{F\} = [k] \cdot \{d\}$$

where $\{F\}$ is the column matrix of member forces.

The inherent nature of the seismic analysis problem, involving time varying ground deformation, necessitates the inclusion of both mass parameters and some allowance for the energy absorbing capacity of the structural system. The lumping of masses at the node points, as outlined in discussing step (1) above, is convenient and reasonably realistic for many civil engineering structures but it is not always possible to justify the frequently used representation of the energy dissipation by an equivalent viscous damping factor. Certainly the manipulation of the mathematical model so devised is relatively easy and, for small values of damping, the concept of equivalent viscous damping is of value[58] but in the study of the seismic response of certain structural systems, including water towers which have yielding braces equivalent viscous damping is an inadequate representation of the energy absorption[59]. Consideration of the actual load/deflection properties of the structure is a necessary prerequisite to valid response predictions of such systems and the mathematical model must include provision for this.

IV.3 Determination of Elastic Dynamic Response of Multi-Mass Systems by Numerical Integration

If consideration of deflections is limited to one degree of freedom only, the equation describing the dynamic equilibrium of a multi-mass system subjected to earthquake ground motion may be

written

$$[M] \circ \{\ddot{x}\} + [C] \circ \{\dot{x}\} + [K]_{LAT} \circ \{x\} = -[M] \circ \ddot{x}_g \qquad \ldots 4(1)$$

where $[M]$ is the diagonal matrix of the lumped masses

$[C]$ is the equivalent viscous damping matrix

$[K]_{LAT}$ is the lateral stiffness matrix of the system i.e. the section of the complete stiffness matrix appropriate to the degree of freedom considered.

$\{x\}$ is the column vector of the mass displacements in the sense of the degree of freedom considered, (measured relative to the base of the structure).

$\ddot{x}_g$ is the base acceleration (i.e. the acceleration of the ground)

$\{\dot{}\}$ derivative with respect to time

$\{\ddot{}\}$ second derivative with respect to time.

This equation may be solved by direct numerical integration using a step-by-step process. Several numerical integration processes may be used. Essentially all involve the calculation of values of displacement, velocity and acceleration of the system at selected time intervals. The selection of an optimum integration step interval must take into account the particular method used as it significantly effects the stability of the process.

Digital computer structural analysis techniques characteristically require substantial allocation of storage to that part of the process which involves the mathematical modelling of the structure. This is

particularly so in investigations of post-elastic behaviour and in many practical cases the computer size critically restricts the adequate representation of the system. Consequently, since the numerical analysis has to be treated as a peripheral problem to that of simulating the structural system it is desirable to use an integration procedure which involves small storage requirements provided that it is adequate for the analysis being undertaken. Primarily for this reason the numerical integration techniques based on the work of Newmark[60] are preferred to other methods such as those of Kutta[61] and Nordsieck[62].

An investigation[63] of various possible variations of the Newmark linear variation in acceleration method was prompted by the necessity to use an IBM 360/44 computer which has a single precision 32 bit word length and operates in hexadecimal arithmetic and it was established that if the integration time step interval is sufficiently small and if sufficient significant figures are used in the calculations, the more involved techniques provide little practical advantage. Consequently the linear variation in acceleration method was used in this investigation. The assumption of linear acceleration variation enables two equations to be set up relating displacement and velocity at time t in terms of the acceleration at time t and the displacement, velocity and acceleration at time $t-\Delta t$ where Δt is a time interval.

From Figure 4.1

$$\ddot{x} = \ddot{x}_{t-\Delta t} + At' \qquad \ldots 4(2)$$

where A is an integration constant.

Integrating with respect to time,

$$\dot{x} = B + \ddot{x}_{t-\Delta t}\, t' + \tfrac{1}{2} At'^2 \qquad \ldots 4(3)$$

where B is an integration constant

at $t' = 0$, $\dot{x} = \dot{x}_{t-\Delta t} = B$

Integrating again with respect to time,

$$x = C + \dot{x}_{t-\Delta t}\, t' + \tfrac{1}{2} \ddot{x}_{t-\Delta t}\, t'^2 + \frac{1}{6} At'^3 \qquad \ldots 4(4)$$

where C is an integration constant

at $t' = 0$, $x = x_{t-\Delta t} = C$

Now at $t' = \Delta t$, $\ddot{x} = \ddot{x}_t$

Therefore $\ddot{x}_t = \ddot{x}_{t-\Delta t} + A\Delta t$

and $A = \dfrac{\ddot{x}_t - \ddot{x}_{t-\Delta t}}{\Delta t}$

Hence substituting for A, B and C and setting $t' = \Delta t$ two equations are obtained, one for velocity

$$\dot{x}_t = \dot{x}_{t-\Delta t} + \frac{\Delta t}{2} \ddot{x}_{t-\Delta t} + \frac{\Delta t}{2} \ddot{x}_t \qquad \ldots 4(5)$$

and the other for displacement

$$x_t = x_{t-\Delta t} + \Delta t\, \dot{x}_{t-\Delta t} + \frac{\Delta t^2}{3} \ddot{x}_{t-\Delta t} + \frac{\Delta t^2}{6} \ddot{x}_t \qquad \ldots 4(6)$$

If these expressions are substituted into the original dynamic equilibrium equation 4(1) an expression for $\{\ddot{x}\}_t$ can be determined. Thus providing that the initial displacement and velocity are postulated and that correct stiffness and damping matrices are available, the numerical integration may be carried out. If it is assumed that the system is initially at rest,

$$\{x\}_o = \{\dot{x}\}_o = 0$$

and the initial acceleration of the system is then equal to the initial exciting acceleration.

The appropriate stiffness matrix may be derived from the overall stiffness matrix $[K]$ which can be assembled by the standard direct stiffness method.

In the mathematical modelling of the elastic system the damping is usually assumed to be of viscous form and the damping matrix $[C]$ may be set up by assuming two sets of dampers, one associated with the springs and the other with the masses.

$$\text{i.e.} \quad [C] = c_r[M] + c_g[K] \qquad \ldots 4(7)$$

where c_r and c_g are damping coefficients.

It may be shown[64] that

$$(c_g \omega_n^2 + c_r)_n = C_n (2\omega_n) \qquad \ldots 4(8)$$

where C_n is the ratio of actual to critical damping in mode n and ω_n is the natural frequency of node n. This equation involves only the natural frequency and consequently the damping

coefficients and proportion of critical damping in each mode are directly related. The two coefficients may be adjusted until a reasonable value is obtained for the damping in each of two modes, in seismic analysis normally the lowest two, providing that the modal frequencies are known. Thus the establishement of the damping matrix requires the prior determination of the modal frequencies. This may be readily undertaken using the system stiffness properties previously discussed in the following manner.

Combining the elastic relation between applied loads $\{L\}$ and distortions $\{d\}$

$$\text{i.e. } \{L\} = [K] \cdot \{d\} \qquad \ldots 4(9)$$

with the harmonic condition

$$\ddot{d} = -\omega_n^2 \{d\} \qquad \ldots 4(10)$$

and substituting in the dynamic equilibrium equation

$$\{L\} = -[M] \{\ddot{d}\} \qquad \ldots 4(11)$$

the expression

$$[M] \cdot \omega_n^2 \cdot \{d\} = [K]_{LAT} \cdot \{d\} \qquad \ldots 4(12)$$

may be obtained.

An iterative solution[65] of this equation will yield the higher mode characteristics first whereas the lower mode properties are of much greater significance in seismic analysis since almost all of the total structural response is contained in the lowest three normal modes.

Consequently the last expression is usually rearranged to give

$$\frac{1}{\omega_n^2} \cdot \{d\} = [K]_{LAT}^{-1} \cdot [M] \cdot \{d\} \qquad \ldots 4(13)$$

and an iterative solution of this will yield the fundamental mode properties first, followed by the higher mode characteristics in ascending order.

In summary then, after the structure has been idealised the analysis procedure described above initially involves the evaluation of the stiffness properties of the system, followed by a modal analysis. The damping characteristics are then determined and, together with the stiffness properties, are used in the numerical integration procedure to determine the elastic dynamic response to the earthquake disturbance. A numerical example of this procedure is presented in Chapter Six.

IV.4 Equations of Motion of A Yielding System

In the subsequent analyses of cross braced frames having yielding members it is assumed that, under seismic loading, the vertical and rotational inertia forces are negligible so that only the horizontal inertia forces are considered.

The equation of dynamic equilibrium of a multimass viscously damped, elastic system subjected to external excitation viz;

$$[M] \cdot \{\ddot{x}\} + [C] \cdot \{\dot{x}\} + [K]_{LAT} \cdot \{x\} = -[M] \cdot \ddot{x}_g \qquad \ldots 4(14)$$

is actually nonlinear in the case of yielding systems because the stiffness is dependent on the magnitude of the response. However when it is assumed that the restoring force remains linear over a short step interval Δt then

$$[M] \circ \{\Delta\ddot{x}\} + [C] \circ \{\Delta\dot{x}\} + [K]_{LAT} \circ \{\Delta x\} = -[M] \circ \Delta\ddot{x}_g \qquad \ldots 4(15)$$

represents the set of linear equations which may be solved by numerical integration techniques.

Then, from equation 4(6),

$$\{\Delta x\} = \Delta t \circ \{\dot{x}\}_{t-\Delta t} + \frac{\Delta t^2}{3} \circ \{\ddot{x}\}_{t-\Delta t} + \frac{\Delta t^2}{6} \circ \{\dddot{x}\}_t$$

$$= \Delta t \circ \{\dot{x}\}_{t-\Delta t} + \frac{\Delta t^2}{2} \circ \{\ddot{x}\}_{t-\Delta t} + \frac{\Delta t^2}{6} \circ \{\Delta\ddot{x}\} \qquad \ldots 4(16)$$

and, from equation 4(5),

$$\{\Delta\dot{x}\} = \frac{\Delta t}{2} \circ \{\ddot{x}\}_{t-\Delta t} + \frac{\Delta t}{2} \circ \{\dddot{x}\}_t$$

$$= \Delta t \circ \{\ddot{x}\}_{t-\Delta t} + \frac{\Delta t}{2} \circ \{\Delta\ddot{x}\} \qquad \ldots 4(17)$$

Solving equation 4(16) for $\{\Delta\ddot{x}\}$ gives

$$\{\Delta\ddot{x}\} = \frac{6}{\Delta t^2} \circ \{\Delta x\} - \frac{6}{\Delta t} \circ \{\dot{x}\}_{t-\Delta t} - 3 \circ \{\ddot{x}\}_{t-\Delta t}$$

$$= \frac{6}{\Delta t^2} \circ \{\Delta x\} + \{A\} \qquad \ldots 4(18)$$

where

$$\{A\} = -\frac{6}{\Delta t} \circ \{\dot{x}\}_{t-\Delta t} - 3 \circ \{\ddot{x}\}_{t-\Delta t} \qquad \ldots 4(19)$$

and substituting in equation 4(17) for $\{\Delta\ddot{x}\}$ gives

$$\{\Delta\dot{x}\} = \Delta t \circ \{\ddot{x}\}_{t-\Delta t} + \frac{3}{\Delta t} \circ \{\Delta x\} - 3 \circ \{\dot{x}\}_{t-\Delta t} - \frac{3}{2} \Delta t \circ \{\ddot{x}\}_{t-\Delta t}$$

$$= \frac{3}{\Delta t} \circ \{\Delta x\} - 3 \circ \{\dot{x}\}_{t-\Delta t} - \frac{\Delta t}{2} \{\ddot{x}\}_{t-\Delta t}$$

$$= \frac{3}{\Delta t} \circ \{\Delta x\} + \{B\} \qquad \ldots 4(20)$$

where

$$\{B\} = -3 \circ \{\dot{x}\}_{t-\Delta t} - \frac{\Delta t}{2} \{\ddot{x}\}_{t-\Delta t} \qquad \ldots 4(21)$$

These expressions for $\{\Delta\dot{x}\}$ and $\{\Delta\ddot{x}\}$ may be substituted in equation 4(15) to give a matrix equation which may be solved for $\{\Delta x\}$

i.e. $$[K^*] \circ \{\Delta x\} = \{\Delta R\} \qquad \ldots 4(22)$$

where $$[K^*] = \left[\frac{6}{\Delta t^2} \circ [M] + \frac{3}{\Delta t} \circ [C] + [K]_{LAT}\right] \qquad \ldots 4(23)$$

and $$\{\Delta R\} = -\left\{\Delta\ddot{x}_g \circ [M] + [M] \circ \{A\} + [C] \circ \{B\}\right\} \qquad \ldots 4(24)$$

The numerical process used involves initialising the values of the relative velocity and displacement vectors to zero, assuming the system is initially at rest. Then

$$\{\ddot{x}_o\} = -\ddot{x}_g \circ \{1\} \qquad \ldots 4(25)$$

and the following steps are executed repeatedly

$$\{A\} = -\frac{6}{\Delta t} \circ \{\dot{x}\} - 3 \circ \{\ddot{x}\}$$

$$\{B\} = -3 \circ \{\dot{x}\} - \frac{\Delta t}{2}\{\ddot{x}\}$$

$$\{\Delta R\} = -\left\{\Delta \ddot{x}_g \circ [M] + [M] \circ \{A\} + [C] \circ \{B\}\right\}$$

$$[K^*] = \left[\frac{6}{\Delta t^2} \circ [M] + \frac{3}{\Delta t} \circ [C] + [K]_{LAT}\right]$$

$$\{\Delta x\} = [K^*]^{-1} \circ \{\Delta R\}$$

$$\{x\} = \{x\} + \{\Delta x\}$$

$$\{\dot{x}\} = \{\dot{x}\} + \frac{3}{\Delta t} \circ \{\Delta x\} + \{B\}$$

$$\{\ddot{x}\} = \{\ddot{x}\} + \frac{6}{\Delta t^2}\{x\} + \{A\}$$

The matrix $[K^*]$ is set up on the first pass and is used repeatedly until the lateral stiffness of the system changes when it is modified to allow for yielding or fractured members.

IV.5 Mathematical Model of an Elevated Water Tank

For the purpose of seismic response calculation an elevated water tank supported on a braced framework can be modelled (see Figure 4.2) as a two mass system (representing the convective and impulsive effects of the water as described in Chapter Two, the mass m_1 being the convective mass, m_o the impulsive mass and m_s the mass of the tank structure), carried on an idealised frame of members whose own mass is lumped at the beam positions (M_1, M_2, M_3 in Figure 4.2). The system shown, in which the elevation of the tower has three braced panels, then becomes one of four discrete masses, M_1, M_2, $M_3 + m_s + m_o$ and m_1

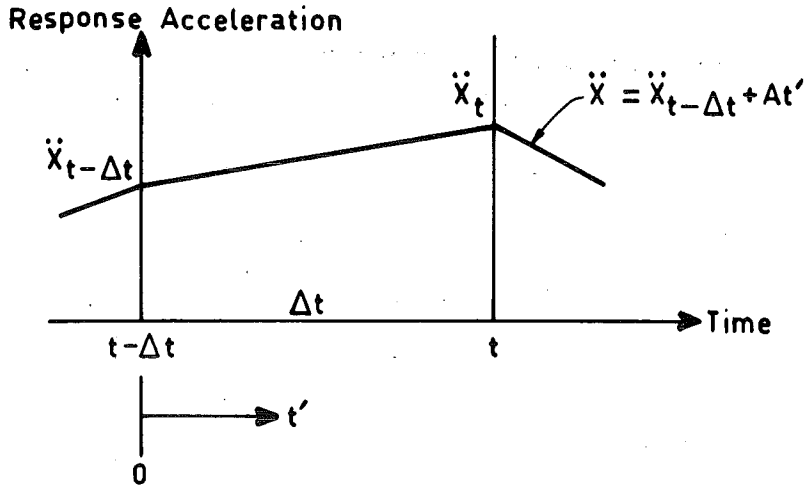

Fig. 4.1 — LINEAR ACCELERATION VARIATION

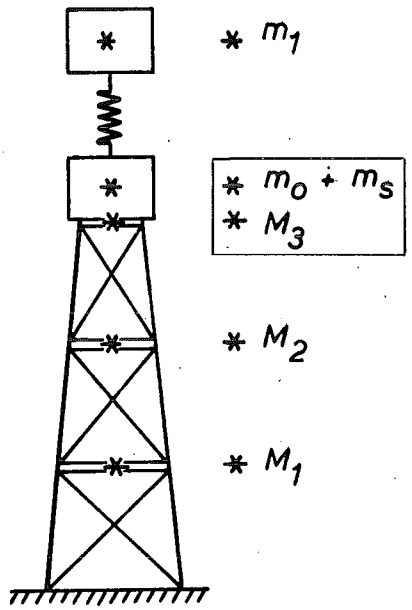

Fig 4.2 — IDEALISED THREE PANEL ELEVATED WATER TANK

connected by links which are essentially elastic but which, in the case of the panel links, have stiffnesses which vary with lateral displacement as the bracing members slacken, yield or fracture.

Equations representing this system may be set up and solved by numerical integration techniques as described earlier in this chapter, using digitised ground accelerations as the excitation, and the displacement response of the structure to earthquakes may thus be determined.

C H A P T E R F I V E

COMPUTER PROGRAMS USED IN THE ANALYSES OF CROSS BRACED FRAMES

V.1 Stiffness Matrix Assembly Program

The determination of the frame stiffness matrices appropriate to the cross braced towers examined was undertaken using the '2-D Structure' program lodged in the University of Canterbury Civil Engineering Department's computer program library. This program enables the joint displacements and element forces in any two dimensional structure composed of an arbitrary assemblage of one and two dimensional elements to be calculated. The stiffness matrix [K] is assembled using the Direct Stiffness Method[66] and the equation $\{L\} = [K]\{d\}$ is solved by first decomposing the stiffness matrix viz:-

$$[K] = [Q][D][Q^T]$$

where [D] is the matrix of pivots

and [Q] is the lower triangular matrix of multipliers.

The solution for the displacements then follows a two step operation[67], first the forward reduction of the applied load vector {L} to give the vector {y}

$$[Q] \cdot [D] \cdot \{y\} = \{L\}$$

followed by the back substitution for the displacement $\{d\}$

$$[Q^T] \cdot \{d\} = \{y\}$$

The particular application of this program to the calculation of a frame stiffness matrix involves the determination of the deformations occurring when unit actions are applied at the node points. A listing of the program, together with typical output and details of the frame analysed, is presented in Appendix IV, section LP 1.

V.2 Normal Mode Properties Program

The normal mode properties of the structures studied were established using the Civil Engineering Department's computer library program 'Normal Mode Properties from F_{LAT}'. This program enables the eigenvalue, eigenvector analysis to be undertaken by an iterative technique[65] using the lateral flexibility and mass properties as input and in addition to calculating the normal mode frequencies and displaced shapes, also computes the 1 g displacements and subsequently the maximum seismic displacements and inter storey shears (assuming the structure to remain fully elastic) resulting from the application of the Skinner response spectrum[68].

In the application of this program to the study of braced frameworks the primary use was to determine the characteristic mode frequencies. A listing of the program, together with typical output is presented in Appendix IV, section LP 2.

V.3 Damping Matrix Formulation Program

Damping coefficients and damping matrices used in subsequent analyses were calculated using the library program 'DAMPING MATRIX'. Based on the application of equation 4(8) (section IV.3) this program enables the damping characteristics of a multimass system to be determined in such a manner that the fractions of critical damping in the first two modes are set at specified values. A list of the program is presented in Appendix IV, section LP 2A.

V.4 Matrix Inversion Program

Since the normal mode properties program requires as input the appropriate lateral flexibility matrix and the stiffness matrix assembly program which was used enabled determination of the lateral stiffness matrix of those frames examined, it was necessary to undertake inversion of some matrices in the course of the analysis procedure adopted. A simple matrix inversion program, based on the Gauss-Jordan elimination technique was used and is listed in Appendix IV, section LP 3.

V.5 Dynamic Elastic Response Program

In order to determine the elastic seismic response for purposes of partially verifying the functioning of the main program used in cross braced frame analysis which is described subsequently the elastic response analysis program ELRES lodged in the Civil Engineering Department's computer library was used.

Developed by Walpole[69] this program uses a numerical integration procedure based on the Newmark constant linear acceleration

method applied to equation 4(1) to calculate the elastic response of a multi-mass system to Berg format earthquake records. Input includes details of the discrete masses, the lateral stiffness matrix, the damping matrix, the selected time step for the integration process and the digitised earthquake record. The output is in the form of a table presenting the displacements of each mass at each time step and a summary of the maximum displacement undergone by each mass and the time at which this occurs. A listing of the program, together with typical output is presented in Appendix IV, section LP 4.

V.6 Cross Braced Frame Dynamic Analysis Program (XBFRME)

The seismic response of an elevated water tank, supported on cross braced towers having bracing with prestressed and yielding characteristics, was calculated using a specially written digital computer program XBFRME. This program consists of a series of subprograms and subroutines, each of which is described separately below. A complete listing of XBFRME is presented in LIST 4.

V.6 (i) KLAT

The first main sub-section of XBFRME consists of a frame lateral matrix assembly program in which particular provision is made for the characteristics of the cross bracing members. A flow chart for KLAT is presented in Figure 5.1 and a listing of a stand-alone version is given in LIST 3.

After setting up arrays and initialising vectors, the program determines the initial stiffness characteristics of the cross bracing

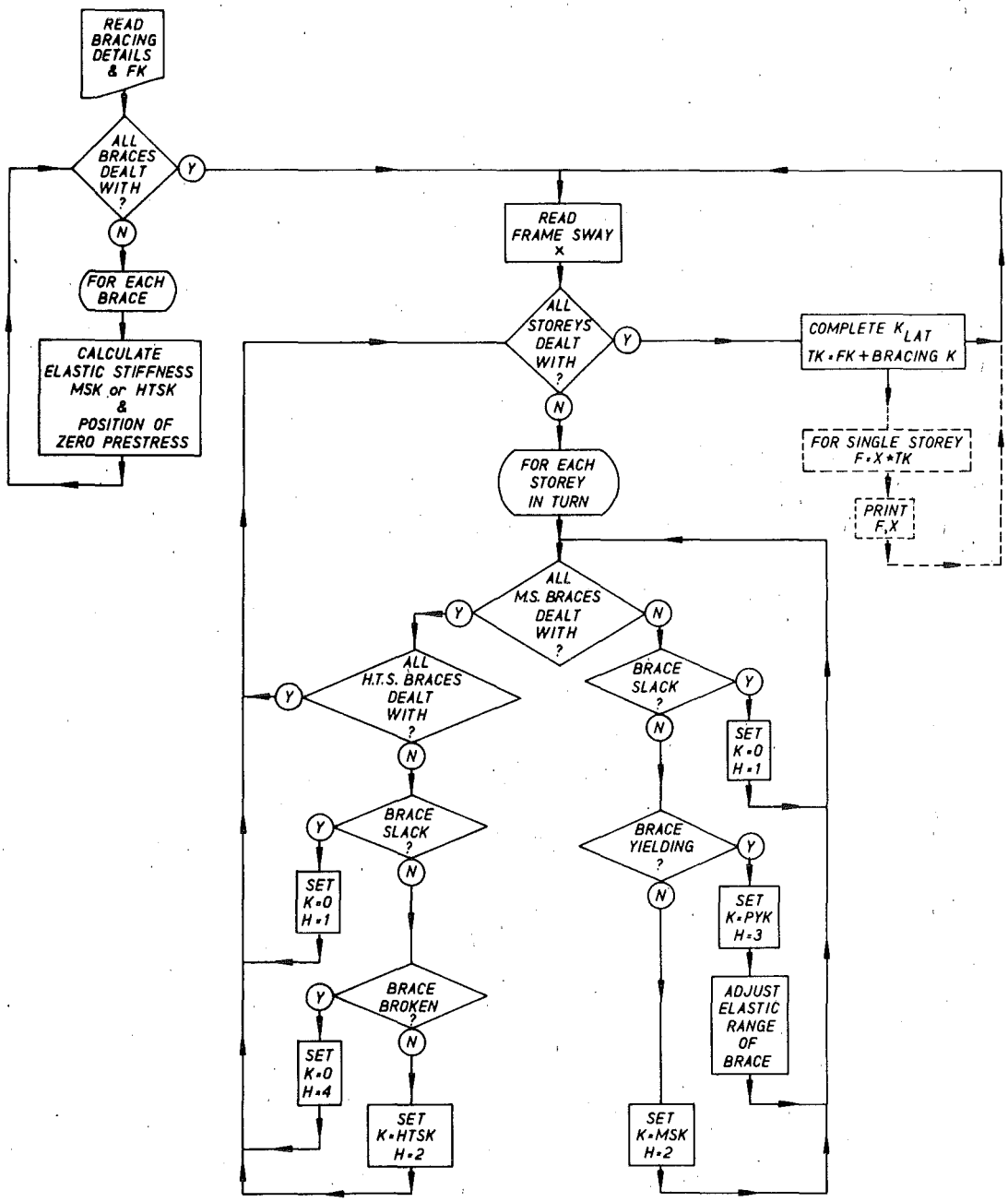

Fig.5.1 : FLOW CHART FOR SUBPROGRAM 'KLAT'

members in a loop which causes the bracing details to be read, the stiffnesses to be computed and the positions of the frame corresponding to the various prestressed members just becoming slack to be calculated.

After reading in the deflected shape of the frame a check is made on each bracing member to determine whether, in the case of the mild steel braces, slackness or yielding has been induced or, in the case of the high tensile steel members, slackness or fracture has been caused. If a mild steel rod has yielded an appropriate adjustment to the effective elastic range of the rod, as defined by the overall frame position, is made. In each case the appropriate stiffness value is selected for each rod, according to its load state, and the history indicator is set.

The total stiffness matrix for the braced frame is formed from the initially read bare frame stiffness, and the bracing rod contributions are determined in this KLAT program. New deflections are read in and the check cycle is repeated to establish whether the new positions of the frame induce slackness, yielding or fracture. When necessary the total frame stiffness is recomputed and the history records are updated. The reading of new deflections is then repeated as after as required and the cycle continued.

For purposes of checking out the validity of the KLAT program an extra instruction, requiring the computation of the sidesway force corresponding to the input sidesway distortion, was added to the program and the values of both force and deflection were printed out at each step. This enabled the results of the mathematical

```
C
C                    SUBPROGRAM KLAT
C                                                                       RS
C     DETERMINATION OF BRACED FRAME STIFFNESS CHARACTERISTICS           RS
C                                                                       RS
      REAL*4 MS1PYK(10)                                                 RS
      REAL*4 MS2PYK(10)                                                 RS
      DIMENSION AMS1K(10),AMS2K(10),FMS1K(10),FMS2K(10),  AMS1P(10),AMS2RS
     1P(10),AMS1Y(10),AMS2Y(10),AMS1DO(10),AMS2DO(10),AMS1DY(10),AMS2DY(RS
     210),AMS1DN(10),AMS2DN(10),HMS1(10),HMS2(10),HTS1K(10),HTS2K(10),HTRS
     3S1P(10),HTS2P(10),HTS1DO(10),HTS2DO(10),HTS1U(10),HTS2U(10),HHTS1(RS
     410),HHTS2(10),HTF(10),HTS1DU(10),HTS2DU(10),FK(10,10),TK(10,10),STRS
     5OREY(10),ANGLE(10),X(10),ADJ(10),F(10),FHTS1K(10),FHTS2K(10),     RS
     6PX(10)                                                            RS
      INTEGER HMS1, HMS2, HHTS1, HHTS2, NS, STOREY, HTF                 RS
      READ(5,100) NS                                                    RS
  100 FORMAT(1015)                                                      RS
C                                                                       RS
C     HISTORY CODE  1 = SLACK, 2 = ELASTIC, 3 = YIELDING, 4 = BROKEN    RS
C     INITIALISE HISTORY CONDITIONS                                     RS
      DO 1 I=1,NS                                                       RS
      HMS1(I)=2                                                         RS
      HMS2(I)=2                                                         RS
      HHTS1(I)=2                                                        RS
      HHTS2(I)=2                                                        RS
    1 HTF(I)=2222                                                       RS
C     INITIALISE KNOWN VECTORS                                          RS
      DO 2 I=1,NS                                                       RS
      X(I)=0.                                                           RS
      PX(I)=0.0                                                         RS
      AMS1DN(I)=0.                                                      RS
      AMS2DN(I)=0.                                                      RS
      DO 2 J=1,NS                                                       RS
    2 TK(I,J)=0.                                                        RS
      AA=0.0                                                            RS
C                                                                       RS
C     READ LATERAL FRAME STIFFNESS MATRIX (K)LAT  ( FOR UNBRACED FRAME )RS
C                                                                       RS
      READ(5,200) ((FK(L,J), L=1,NS),J=1,NS)                            RS
  200 FORMAT(10G10.3)                                                   RS
      WRITE(6,201)                                                      RS
  201 FORMAT(1H1,55X,21HINPUT DATA ECHO CHECK/56X,21(1H-)///            RS
     130X,39HUNBRACED FRAME LATERAL STIFFNESS MATRIX/)                  RS
      WRITE(6,202) ((FK(L,J),L=1,NS),J=1,NS)                            RS
  202 FORMAT(50X,F12.6)                                                 RS
C                                                                       RS
C     READ BRACING DETAILS AND COMPLTE BRACED FRAME STIFFNESS           RS
C                                                                       RS
      DO 3 I=1,NS                                                       RS
C     BRACING MILD STEEL ONE  ( BOTTOM LEFT TO TOP RIGHT)               RS
C     BRACING MILD STEEL TWO  ( TOP LEFT TO BOTTOM RIGHT)               RS
      READ(5,200) A,ZL,AMS1P(I),AMS1Y(I),MS1PYK(I),ANG,EMS              RS
      WRITE(6,203)                                                      RS
  203 FORMAT(1H1,60X,15HBRACING DETAILS/)                               RS
      WRITE(6,200) A,ZL,AMS1P(I),AMS1Y(I),MS1PYK(I),ANG,EMS             RS
      ANG=ANG*3.1415926/180.                                            RS
      AMS1K(I)=EMS*A/ZL*COS(ANG)*COS(ANG)                               RS
      READ(5,200) A,ZL,AMS2P(I),AMS2Y(I),MS2PYK(I),ANG,EMS              RS
      WRITE(6,200) A,ZL,AMS2P(I),AMS2Y(I),MS2PYK(I),ANG,EMS             RS
      ANG=ANG*3.1415926/180.                                            RS
      AMS2K(I)=EMS*A/ZL*COS(ANG)*COS(ANG)                               RS
C     BRACING HIGH TENSILE STEEL ONE  ( BOTTOM LEFT TO TOP RIGHT )      RS
C     BRACING HIGH TENSILE STEEL TWO  ( TOP LEFT TO BOTTOM RIGHT )      RS
      READ(5,200) A,ZL,HTS1P(I),HTS1U(I),ANG,EHT                        RS
      WRITE(6,200) A,ZL,HTS1P(I),HTS1U(I),ANG,EHT                       RS
      ANG=ANG*3.1415926/180.                                            RS
      HTS1K(I)=EHT*A/ZL*COS(ANG)*COS(ANG)                               RS
      READ(5,200) A,ZL,HTS2P(I),HTS2U(I),ANG,EHT                        RS
      WRITE(6,200) A,ZL,HTS2P(I),HTS2U(I),ANG,EHT                       RS
      ANG=ANG*3.1415926/180.                                            RS
      HTS2K(I)=EHT*A/ZL*COS(ANG)*COS(ANG)                               RS
C     ADJUST FOR LOSS OF PRESTRESS CR FOR YIELDING IF NECESSARY         RS
      AMS1DO(I)=-AMS1P(I)/AMS1K(I)                                      RS
      AMS2DO(I)=+AMS2P(I)/AMS2K(I)                                      RS
      HTS1DO(I)=-HTS1P(I)/HTS1K(I)                                      RS
      HTS2DO(I)=+HTS2P(I)/HTS2K(I)                                      RS
      AMS1DN(I)=AMS1DO(I)                                               RS
      AMS2DN(I)=AMS2DO(I)                                               RS
      FMS1K(I)=AMS1K(I)                                                 RS
      FMS2K(I)=AMS2K(I)                                                 RS
      FHTS1K(I)=HTS1K(I)                                                RS
      FHTS2K(I)=HTS2K(I)                                                RS
    3 CONTINUE                                                          RS
C                                                                       RS
C     ADJUST FOR LOSS OF PRESTRESS CR FOR YIELDING IN M.S. IF NECESSARY RS
C     ADJUST FOR LOSS OF PRESTRESS CF FOR FRACTURE IN H.T.S. CITTC      RS
C     SWAY OF TOP OF BRACED PANEL TO RIGHT RELATIVE TO BASE             RS
C     IS POSITIVE X                                                     RS
C     THE INTERSTOREY DRIFT EQUALS X(I) MINUS X(I-1)                    RS
C                                                                       RS
C     MILD STEEL BRACING ONE                                            RS
C     FIRST, LOSS OF PRESTRESS DUE TO BRACING GOING SLACK               RS
    7 READ(5,200) (X(I),I=1,NS)                                         RS
      DO 5 I=1,NS                                                       RS
      XIO=X(I-1)                                                        RS
      IF(I.EQ.1)XIO=0                                                   RS
      AMS1K(I)=FMS1K(I)                                                 RS
      HMS1(I)=2                                                         RS
      B=X(I)-XIC-AMS1DN(I)                                              RS
      IF(B.GE.0.0) GO TO 10                                             RS
      AMS1K(I)=0.0                                                      RS
      HMS1(I)=1                                                         RS
   10 CONTINUE                                                          RS
C     SECOND QUERY IF YIELD HAS TAKEN PLACE                             RS
      AMS1DY(I)=(AMS1Y(I)/FMS1K(I))+AMS1DN(I)                           RS
      C=X(I)-XIC-AMS1DY(I)                                              RS
      IF(C.LT.0.0) GO TO 20                                             RS
      AMS1K(I)=MS1PYK(I)                                                RS
      HMS1(I)=3                                                         RS
C     FOLLOWING YIELD ADJUSTMENT OF ELASTIC RANGE POSITION NECESSARY    RS
      ADJ(I)=X(I)-XIO-AMS1DY(I)                                         RS
      AMS1DN(I)=AMS1DN(I)+ADJ(I)                                        RS
   20 CONTINUE                                                          RS
      WRITE(6,300)AMS1K(I)                                              RS
  300 FORMAT(70X,G10.3)                                                 RS
C     MILD STEEL BRACING TWO                                            RS
      AMS2K(I)=FMS2K(I)                                                 RS
      HMS2(I)=2                                                         RS
      BB=X(I)-XIO-AMS2DN(I)                                             RS
```

LIST 3

```
      IF(BB.LE.0.0) GO TO 30                                            RS
      AMS2K(I)=0.0                                                      RS
      HMS2(I)=1                                                         RS
   30 CONTINUE                                                          RS
      AMS2DY(I)=(-AMS2Y(I)/FMS2K(I))+AMS2DN(I)                          RS
      CC=X(I)-XIO-AMS2DY(I)                                             RS
      IF(CC.GT.0.0) GO TO 40                                            RS
      AMS2K(I)=MS2PYK(I)                                                RS
      HMS2(I)=3                                                         RS
      ADJ(I)=X(I)-XIC-AMS2DY(I)                                         RS
      AMS2DN(I)=AMS2DN(I)+ADJ(I)                                        RS
   40 CONTINUE                                                          RS
      WRITE(6,300)AMS2K(I)
C     HIGH TENSILE STEEL BRACING CNE                                    RS
      IF(HHTS1(I).GT.3) GO TO 60                                        RS
      HTS1K(I)=FHTS1K(I)                                                RS
      HHTS1(I)=2                                                        RS
      BBB=X(I)-XIO-HTS1DO(I)                                            RS
      IF(BBB.GE.0.0) GO TO 50                                           RS
      HTS1K(I)=0.0                                                      RS
      HHTS1(I)=1                                                        RS
   50 CONTINUE                                                          RS
      HTS1DU(I)=(HTS1U(I)/FHTS1K(I))+HTS1DO(I)                          RS
      CCC=X(I)-XIO-HTS1DU(I)                                            RS
      IF(CCC.LE.0.0) GO TO 60                                           RS
      HTS1K(I)=0.0                                                      RS
      HHTS1(I)=4                                                        RS
   60 CONTINUE                                                          RS
      WRITE(6,300)HTS1K(I)
C     HIGH TENSILE STEEL BRACING TWC                                    RS
      IF(HHTS2(I).GT.3) GO TO 80                                        RS
      HTS2K(I)=FHTS2K(I)                                                RS
      HHTS2(I)=2                                                        RS
      BBBB=X(I)-XIO-HTS2DO(I)                                           RS
      IF(BBBB.LE.0.0) GO TO 70                                          RS
      HTS2K(I)=0.0                                                      RS
      HHTS2(I)=1                                                        RS
   70 CONTINUE                                                          RS
      HTS2DU(I)=(-HTS2U(I)/FHTS2K(I))+HTS2DO(I)                         RS
      CCCC=X(I)-XIO-HTS2DU(I)                                           RS
      IF(CCCC.GE.0.0) GO TO 80                                          RS
      HTS2K(I)=0.0                                                      RS
      HHTS2(I)=4                                                        RS
   80 CONTINUE                                                          RS
      WRITE(6,300)HTS2K(I)
C                                                                       RS
C     FORM TOTAL STIFFNESS MATRIX                                       RS
C                                                                       RS
      DO 4 L=1,NS                                                       RS
      DO 4 J=1,NS                                                       RS
    4 TK(L,J)=FK(L,J)                                                   RS
      TK(1,1)=TK(1,1)+AMS1K(1)+AMS2K(1)+HTS1K(1)+HTS2K(1)               RS
      IF(NS.EQ.1) GO TO 5                                               RS
      DO 6 L=2,NS                                                       RS
      AA=AMS1K(L)+AMS2K(L)+HTS1K(L)+HTS2K(L)                            RS
      TK(L-1,L-1)=TK(L-1,L-1)+AA                                        RS
      TK(L-1,L)=TK(L-1,L)-AA                                            RS
      TK(L,L-1)=TK(L,L-1)-AA                                            RS
    6 TK(L,L)=TK(L,L)+AA                                                RS
    5 CONTINUE                                                          RS
      WRITE(6,200)TK
      DO 210 I=1,NS                                                     RS
      F(I)=F(I)+TK(L,J)*(X(I)-PX(I))                                    RS
      PX(I)=X(I)                                                        RS
  210 HTF(I)=1000*HMS1(I)+100*HMS2(I)+10*FHTS1(I)+HHTS2(I)              RS
  204 FORMAT(2F12.6,I10)                                                RS
      WRITE(6,204) (F(I),I=1,NS),(X(I),I=1,NS),(HTF(I),I=1,NS)          RS
      GO TO 7                                                           RS
      END                                                               RS
```

LIST 3

(continued)

model analysis undertaken using KLAT to be compared directly with the experiments conducted on the single bay braced frames which are reported in Chapter Three.

V.6 (ii) Results of validity check on KLAT

A comparison of the results determined using KLAT with those obtained in a static sidesway stiffness test on a pinned knee frame braced with pairs of diagonally opposed prestressed mild steel and high tensile steel rods is presented in Figure 5.2. To assist identification the curves were deliberately separated by setting the yield level of the mild steel rods 200 lbs lower, in the mathematical analysis, than the true value. This effect is cumulative so that the traces tend to diverge as the loading cycle progresses but it is evident that the mathematical model adequately describes the cross braced framework, particularly in the increasing-load sections of the curves. The slackness which was evident in the experimental frame testing is difficult to represent in the mathematical model and the fact that this is not allowed for in the program accounts for the divergence in the curves on the decreasing-load sections of the cycles. Nevertheless the similarity in the shapes and in particular the slopes, serves to verify the use of the model set up in KLAT as a reasonable representation of the cross braced frame tested.

Some limitations of the mathematical model were examined and the results are presented diagrammatically in Figures 5.3 and 5.4. In analysing the rigid knee braced frame (Figure 5.3) the full curve

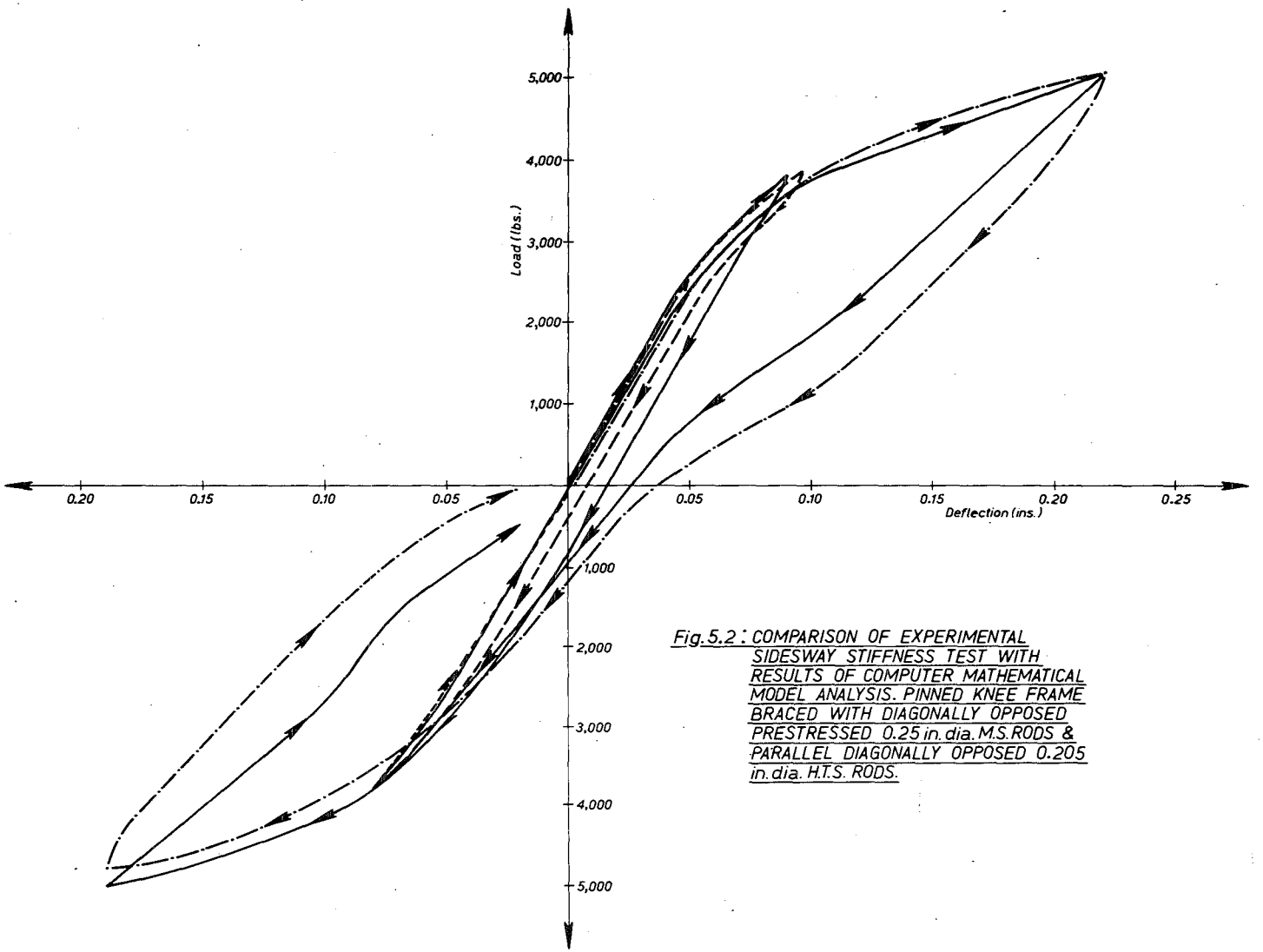

Fig. 5.2 : COMPARISON OF EXPERIMENTAL SIDESWAY STIFFNESS TEST WITH RESULTS OF COMPUTER MATHEMATICAL MODEL ANALYSIS. PINNED KNEE FRAME BRACED WITH DIAGONALLY OPPOSED PRESTRESSED 0.25 in. dia. M.S. RODS & PARALLEL DIAGONALLY OPPOSED 0.205 in. dia. H.T.S. RODS.

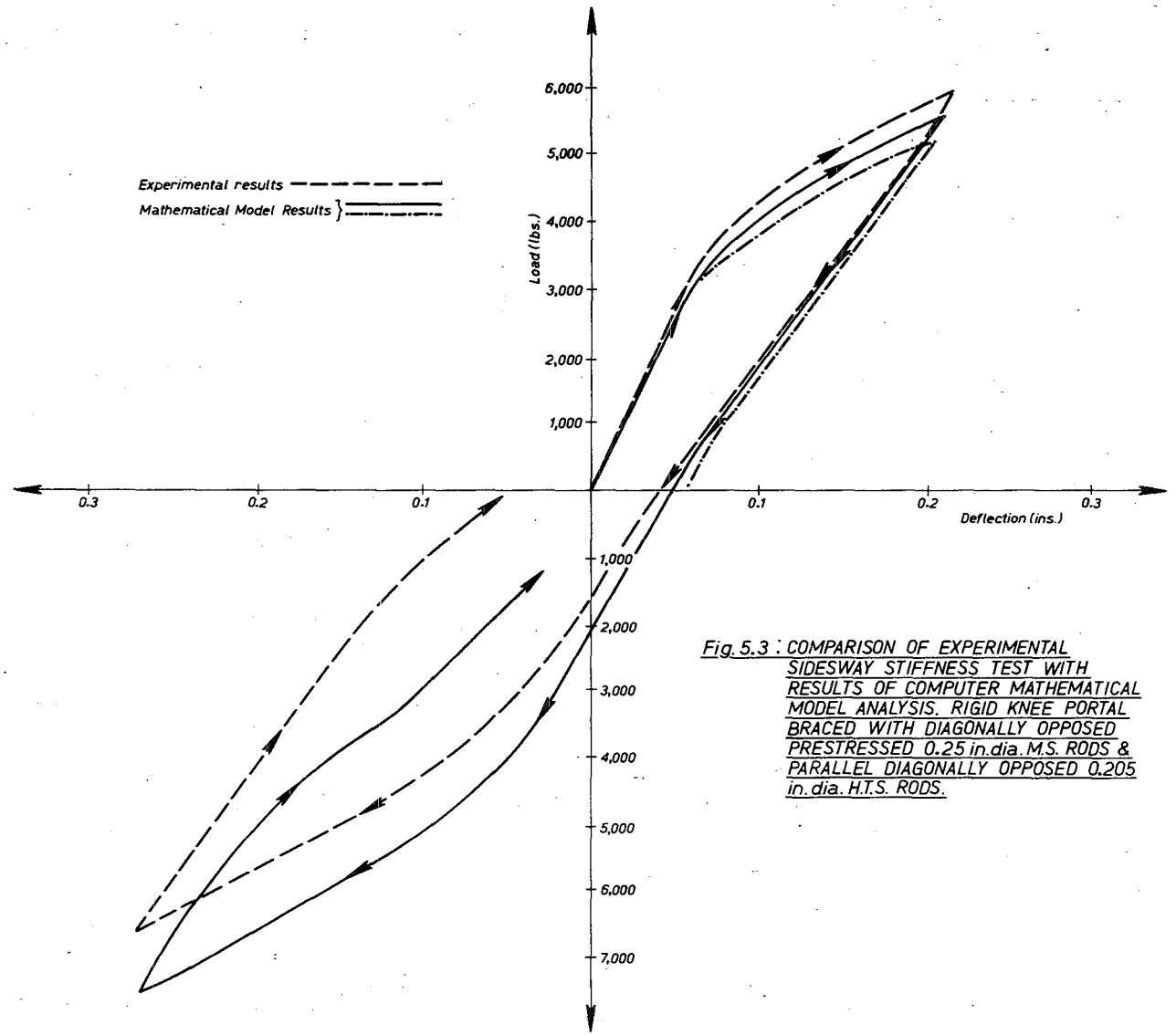

Fig. 5.3 : COMPARISON OF EXPERIMENTAL SIDESWAY STIFFNESS TEST WITH RESULTS OF COMPUTER MATHEMATICAL MODEL ANALYSIS. RIGID KNEE PORTAL BRACED WITH DIAGONALLY OPPOSED PRESTRESSED 0.25 in.dia. M.S. RODS & PARALLEL DIAGONALLY OPPOSED 0.205 in.dia. H.T.S. RODS.

was obtained using a reduced yield level (to separate the curve from the experimental one) and a displacement step interval of 0.03 inch. The curve denoted by crosses was obtained using identical data but with a displacement step interval of 0.01 inch. The effect of the step interval being large is to increase the apparent load on the system along those sections of the curve where the stiffness is in fact falling off, since the curve is built up progressively by projecting across the step interval at a gradient defined by the last value of stiffness. It was not expected that the displacement step interval would be large enough to cause significant errors in the dynamic analysis since the magnitude of the displacement increment is limited by the small integration time step but the result of this preliminary investigation emphasised the need to avoid large step intervals.

The comparison of results obtained in another of the experimental sidesway stiffness tests described in Chapter Three with those obtained using the KLAT program drew attention to another limitation of the mathematical model. As is shown in Figure 5.4, the model does not adequately represent the braced frame beyond about 0.8 in. sidesway. This is undoubtedly because the assumption of a constant post yield stiffness for the remaining active bracing rod becomes invalid at this level of sidesway. Provision could have been made in the program for suitable modification of the element stiffness values at this order of deflection but since it was felt that inter-storey sways of this proportion were generally unacceptable for reasons of instability and other generally neglected, second order effects, an

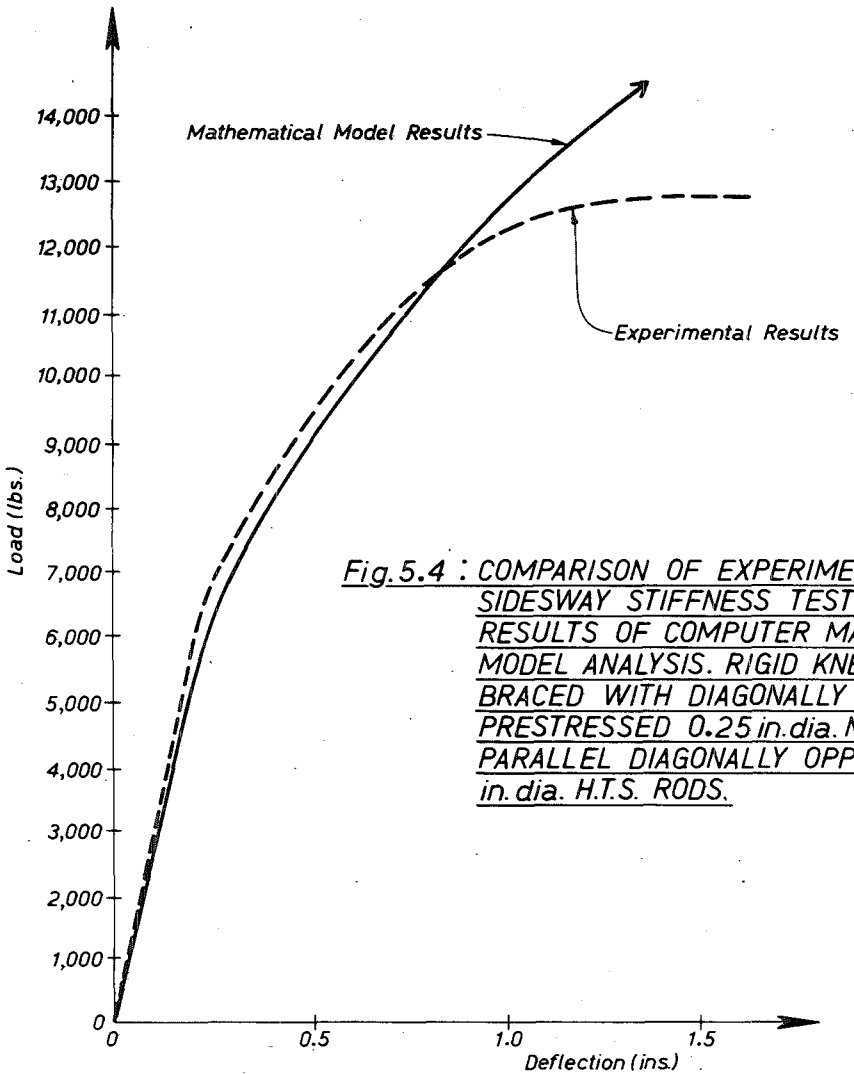

Fig.5.4 : COMPARISON OF EXPERIMENTAL SIDESWAY STIFFNESS TEST WITH RESULTS OF COMPUTER MATHEMATICAL MODEL ANALYSIS. RIGID KNEE PORTAL BRACED WITH DIAGONALLY OPPOSED PRESTRESSED 0.25 in.dia. M.S. RODS & PARALLEL DIAGONALLY OPPOSED 0.205 in. dia. H.T.S. RODS.

appropriate modification to the program was not considered necessary although this upper limit to the validity of the mathematical model clearly exists.

V.6 (iii) Main program XBFRME

A flow chart for program XBFRME is presented in Figure 5.2 and a complete listing is given in LIST 4.

In writing this program considerable effort was expended in an attempt to make it easy to follow, specifically by the generous use of comment cards, but an outline of the main features is included in the following stages of this section of this thesis.

After setting the DIMENSIONS and specifying DOUBLE PRECISION for the variables occurring in the numerical integration portion a control card, which also contains the damping coefficients, is read. Next, details of the mass and stiffness of the unbraced tower are read. After computation of the damping matrix the program essentially duplicates KLAT (LIST 3) until the lateral stiffness matrix of the braced frame has been determined.

The subsequent section contains the numerical integration process which is based on the solution of equation 4(15) by the stepwise process presented at the end of section IV.4. Provision is made for diversion past unnecessary steps in the solution process where these are avoidable, specifically by checking whether the stiffness has changed since the previous cycle and acting accordingly.

The final section of the main program XBFRME enables both written and punched card output to be produced, the selection being made by

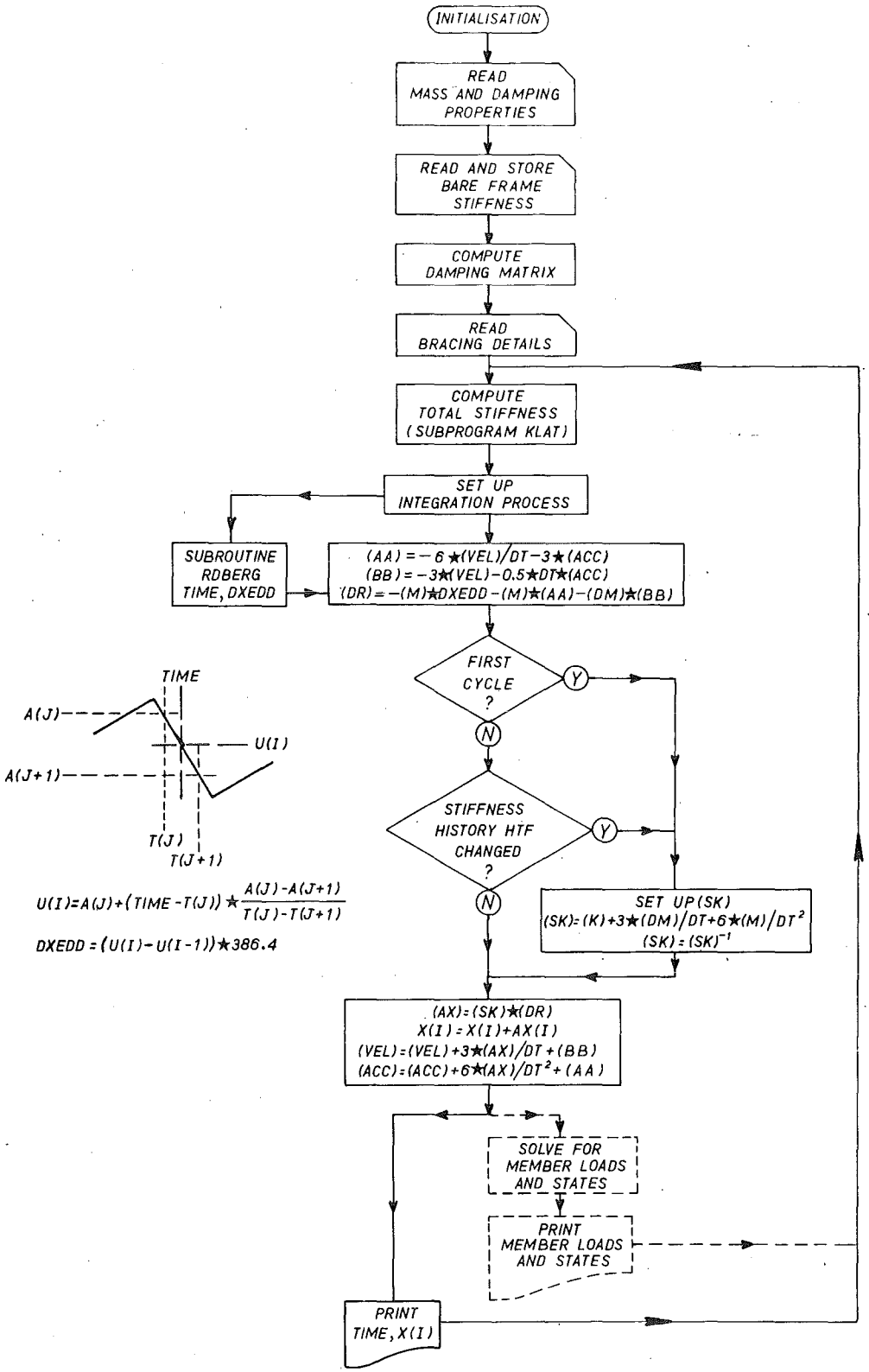

Fig. 5.5 : FLOW CHART FOR COMPUTER PROGRAM XBFRME

```
C
C     ****************************************************************
C     *                                                              *
C     *                      PROGRAM XBFRME                          *
C     *                                                              *
C     *          THIS PROGRAM EXECUTES THE DYNAMIC ANALYSIS OF       *
C     *                                                              *
C     *          SEISMICALLY LOADED CROSS BRACED FRAMEWORKS BY       *
C     *                                                              *
C     *        NUMERICAL INTEGRATION OF THE EQUATIONS OF MOTION      *
C     *                                                              *
C     ****************************************************************
C     *                                                              *
C                                                                     RS
C     GENERAL EQUATION FORM                                           RS
C
C                    (M)(DXDD)+(C)(DXD)+(K)(DX)=-(M)DXEDD             RS
C
C            WHERE M = MASS                                           RS
C               DXDD = DELTA X DOUBLE DOT                             RS
C                  C = DAMPING                                        RS
C                  K = STIFFNESS                                      RS
C                 DX = X DOT                                          RS
C              DXEDD = DELTA GROUND ACCELERATION                      RS
C             X,XD,XDD DESCRIBE MOTION RELATIVE TO GROUND             RS
C             XEDD IS ABSOLUTE GROUND ACCELERATION                    RS
C                                                                     RS
C     *                                                              *
C     ****************************************************************
C
      REAL*4 MS1PYK(10)                                               RS
      REAL*4 MS2PYK(10)                                               RS
      DIMENSION TK(20,20),W(20),DM(20,20),ZM(20),AA(20),VEL(20),BB(20), RS
     1ACC(20),ADJ1(10),BC(20),SK(20,20),DR(20),AX(20),X(20),HTF(10), TXRS
     2(20),BX(20),AMS1K(10),AMS2K(10),FMS1K(10),FMS2K(10),AMS1P(10),AMS2RS
     3P(10),AMS1Y(10),AMS2Y(10),AMS1DO(10),AMS2DO(10),AMS1DY(10),AMS2DY(RS
     410),AMS1DN(10),AMS2DN(10),HMS1(10),HMS2(10),HTS1K(10),HTS2K(10),HTRS
     5S1P(10),HTS2P(10),HTS1DO(10),HTS2DO(10),HTS1U(10),HTS2U(10),HHTS1(RS
     610),HHTS2(10),HTS1DU(10),HTS2DU(10),FK(10,10),STOREY(10),F(10),   RS
     7ANGLE(10),FHTS1K(10),FHTS2K(10),PX(10),PHTF(10),ADJ2(10),TADJ1(10RS
     8),TADJ2(10),XP1(1000),XP2(1000),XP3(1000)                       RS
      DOUBLE PRECISION DT,DR,VEL,ACC,CB,CM,ZM,SK,AX,AA,BB,D           RS
      INTEGER HMS1, HMS2, HHTS1, FHTS2, NM, STOREY, HTF               RS
      COMMON DT                                                       RS
      KC=C                                                            RS
      NXP=0                                                           RS
      NFLAG=0                                                         RS
      READ(5,123) NM,NPS,TR,DT,CG,CR,NPLOT,NWRTE                      RS
  123 FORMAT(2I5,4G10.0,2I5)                                          RS
C                                                                     RS
      WRITE(6,201)                                                    RS
  201 FORMAT(1H1,55X,21HINPUT DATA ECHO CHECK/56X,21(1H-)///)         RS
C                                                                     RS
C     READ MASS MATRIX (ACTUALLY IN WEIGHT UNITS)                     RS
      READ(5,202) (W(I),I=1,NM)                                       RS
  202 FORMAT(8E10.4)                                                  RS
      WRITE(6,203)                                                    RS
  203 FORMAT('0',30X,'MASS MATRIX'/)                                  RS
      WRITE(6,204) (W(I),I=1,NM)                                      RS
  204 FORMAT(20X,8E13.4/)                                             RS
      DO 209 I=1,NM                                                   RS
  209 ZM(I)=W(I)/386.4                                                RS
C                                                                     RS
C     READ LATERAL FRAME STIFFNESS MATRIX (K)LAT  ( FOR UNBRACED FRAME )RS
      READ(5,220) ((FK(L,J), L=1,NM),J=1,NM)                          RS
  220 FORMAT(8G10.3)                                                  RS
      WRITE(6,225)                                                    RS
  225 FORMAT('0',30X,39HUNBRACED FRAME LATERAL STIFFNESS MATRIX/)     RS
      WRITE(6,229) ((FK(L,J),L=1,NM),J=1,NM)                          RS
  229 FORMAT(50X,E12.6)                                               RS
C                                                                     RS
C     COMPUTE DAMPING MATRIX                                          RS
      DO 231 I=1,NM                                                   RS
      DM(I,I)=CR*ZM(I)                                                RS
      DO 230 JI=1,NM                                                  RS
  230 DM(I,JI)=DM(I,JI)+CG*FK(I,JI)                                   RS
  231 CONTINUE                                                        RS
      WRITE(6,232)                                                    RS
  232 FORMAT('0',30X,'DAMPING MATRIX'/)                               RS
      WRITE(6,229)((DM(I,JI),JI=1,NM),I=1,NM)                         RS
C                                                                     RS
C     DETERMINATION OF BRACED FRAME STIFFNESS CHARACTERISTICS         RS
C     HISTORY CODE  1 = SLACK, 2 = ELASTIC, 3 = YIELDING, 4 = BROKEN  RS
C     INITIALISE HISTORY CONDITIONS                                   RS
      DO 1 I=1,NM                                                     RS
      HMS1(I)=2                                                       RS
      HMS2(I)=2                                                       RS
      HHTS1(I)=2                                                      RS
      HHTS2(I)=2                                                      RS
    1 HTF(I)=2222                                                     RS
C     INITIALISE KNOWN VECTORS                                        RS
      DO 2 I=1,NM                                                     RS
      X(I)=0.                                                         RS
      PX(I)=0.0                                                       RS
      AMS1DN(I)=0.                                                    RS
      AMS2DN(I)=0.                                                    RS
      DO 2 J=1,NM                                                     RS
      ADJ1(I)=0                                                       RS
      ADJ2(I)=0                                                       RS
      TADJ1(I)=0.                                                     RS
      TADJ2(I)=0                                                      RS
    2 TK(I,J)=0.                                                      RS
      AK=0.0                                                          RS
C                                                                     RS
C     READ BRACING DETAILS AND COMPUTE BRACED FRAME STIFFNESS         RS
C                                                                     RS
      DO 3 I=1,NM                                                     RS
C     BRACING MILD STEEL ONE  ( BOTTOM LEFT TO TOP RIGHT)             RS
C     BRACING MILD STEEL TWO  ( TOP LEFT TO BOTTOM RIGHT)             RS
      READ(5,220) A,ZL,AMS1P(I),AMS1Y(I),MS1PYK(I),ANG,EMS            RS
      WRITE(6,233)                                                    RS
  233 FORMAT(1H1,60X,15HBRACING DETAILS/)                             RS
      WRITE(6,220) A,ZL,AMS1P(I),AMS1Y(I),MS1PYK(I),ANG,EMS           RS
      ANG=ANG*3.1415926/180.                                          RS
      AMS1K(I)=EMS*A/ZL*COS(ANG)*COS(ANG)                             RS
      READ(5,220) A,ZL,AMS2P(I),AMS2Y(I),MS2PYK(I),ANG,EMS            RS
      WRITE(6,220) A,ZL,AMS2P(I),AMS2Y(I),MS2PYK(I),ANG,EMS           RS
      ANG=ANG*3.1415926/180.                                          RS
      AMS2K(I)=EMS*A/ZL*COS(ANG)*COS(ANG)                             RS
C     BRACING HIGH TENSILE STEEL ONE  ( BOTTOM LEFT TO TOP RIGHT )    RS
```

LIST 4

```
C     BRACING HIGH TENSILE STEEL TWC  ( TOP LEFT TO BOTTOM RIGHT )      RS
      READ(5,220) A,ZL,HTS1P(I),HTS1U(I),ANG,EHT                        RS
      WRITE(6,220) A,ZL,HTS1P(I),HTS1U(I),ANG,EHT                       RS
      ANG=ANG*3.1415926/180.                                            RS
      HTS1K(I)=EHT*A/ZL*COS(ANG)*CCS(ANG)                               RS
      READ(5,220) A,ZL,HTS2P(I),HTS2U(I),ANG,EHT                        RS
      WRITE(6,220) A,ZL,HTS2P(I),HTS2U(I),ANG,EHT                       RS
      ANG=ANG*3.1415926/180.                                            RS
      HTS2K(I)=EHT*A/ZL*CCS(ANG)*CCS(ANG)                               RS

C     ADJUST FOR LOSS OF PRESTRESS OR FOR YIELDING IF NECESSARY         RS
      AMS1DO(I)=-AMS1P(I)/AMS1K(I)                                      RS
      AMS2DO(I)=+AMS2P(I)/AMS2K(I)                                      RS
      HTS1DO(I)=-HTS1P(I)/HTS1K(I)                                      RS
      HTS2DO(I)=+HTS2P(I)/HTS2K(I)                                      RS
      AMS1DN(I)=AMS1DO(I)                                               RS
      AMS2DN(I)=AMS2DO(I)                                               RS
      FMS1K(I)=AMS1K(I)                                                 RS
      FMS2K(I)=AMS2K(I)                                                 RS
      FHTS1K(I)=HTS1K(I)                                                RS
      FHTS2K(I)=HTS2K(I)                                                RS
    3 CONTINUE                                                          RS
C                                                                       RS
C     INITIALISE INTEGRATION PROCESS                                    RS
C                                                                       RS
      IF(DT.EQ.0.) DT=1.0/256.0                                         RS
      DO235 I=1,NM                                                      RS
      DR(I)=0.                                                          RS
      X(I)=0.                                                           RS
      VEL(I)=0.                                                         RS
  235 ACC(I)=0.                                                         RS
C                                                                       RS
C     ADJUST FOR LOSS OF PRESTRESS OR FOR YIELDING IN M.S. IF NECESSARY RS
C     ADJUST FOR LOSS OF PRESTRESS OF FOR FRACTURE IN H.T.S. DITTO      RS
C     SWAY OF TOP OF BRACED PANEL TO RIGHT RELATIVE TO BASE             RS
C     IS POSITIVE X                                                     RS
C     THE INTERSTOREY DRIFT EQUALS X(I) MINUS X(I-1)                    RS
C                                                                       RS
C     MILD STEEL BRACING ONE                                            RS
C     FIRST, LOSS OF PRESTRESS DUE TO BRACING GOING SLACK               RS
  240 DO 5 I=1,NM                                                       RS
      XIO=X(I-1)                                                        RS
      IF(I.EQ.1)XIO=0                                                   RS
      AMS1K(I)=FMS1K(I)                                                 RS
      HMS1(I)=2                                                         RS
      B=X(I)-XIO-AMS1DN(I)                                              RS
      IF(B.GE.0.0) GO TO 10                                             RS
      AMS1K(I)=0.0                                                      RS
      HMS1(I)=1                                                         RS
   10 CONTINUE                                                          RS
C     SECOND QUERY IF YIELD HAS TAKEN PLACE                             RS
      AMS1DY(I)=(AMS1Y(I)/FMS1K(I))+AMS1DN(I)                           RS
      C=X(I)-XIO-AMS1DY(I)                                              RS
      IF(C.LT.0.0) GO TO 20                                             RS
      AMS1K(I)=MS1PYK(I)                                                RS
      HMS1(I)=3                                                         RS
C     FOLLOWING YIELD ADJUSTMENT OF ELASTIC RANGE POSITION NECESSARY    RS
      ADJ1(I)=X(I)-XIO-AMS1DY(I)                                        RS
      AMS1DN(I)=AMS1DN(I)+ADJ1(I)                                       RS
      TADJ1(I)=TADJ1(I)+ADJ1(I)                                         RS

   20 CONTINUE                                                          RS
C     MILD STEEL BRACING TWO                                            RS
      AMS2K(I)=FMS2K(I)                                                 RS
      HMS2(I)=2                                                         RS
      GG=X(I)-XIO-AMS2DN(I)                                             RS
      IF(GG.LE.0.0) GO TO 30                                            RS
      AMS2K(I)=0.0                                                      RS
      HMS2(I)=1                                                         RS
   30 CONTINUE                                                          RS
      AMS2DY(I)=(-AMS2Y(I)/FMS2K(I))+AMS2DN(I)                          RS
      CC=X(I)-XIO-AMS2DY(I)                                             RS
      IF(CC.GT.0.0) GO TO 40                                            RS
      AMS2K(I)=MS2PYK(I)                                                RS
      HMS2(I)=3                                                         RS
      ADJ2(I)=X(I)-XIO-AMS2DY(I)                                        RS
      AMS2DN(I)=AMS2DN(I)+ADJ2(I)                                       RS
      TADJ2(I)=TADJ2(I)+ADJ2(I)                                         RS
   40 CONTINUE                                                          RS
C     HIGH TENSILE STEEL BRACING ONE                                    RS
      IF(HHTS1(I).GT.3) GO TO 60                                        RS
      HTS1K(I)=FHTS1K(I)                                                RS
      HHTS1(I)=2                                                        RS
      BBB=X(I)-XIO-HTS1DO(I)                                            RS
      IF(BBB.GE.0.0) GO TO 50                                           RS
      HTS1K(I)=0.0                                                      RS
      HHTS1(I)=1                                                        RS
   50 CONTINUE                                                          RS
      HTS1DU(I)=(HTS1U(I)/FHTS1K(I))+HTS1DO(I)                          RS
      CCC=X(I)-XIO-HTS1DU(I)                                            RS
      IF(CCC.LE.0.0) GO TO 60                                           RS
      HTS1K(I)=0.0                                                      RS
      HHTS1(I)=4                                                        RS
   60 CONTINUE                                                          RS
C     HIGH TENSILE STEEL BRACING TWO                                    RS
      IF(HHTS2(I).GT.3) GO TO 80                                        RS
      HTS2K(I)=FHTS2K(I)                                                RS
      HHTS2(I)=2                                                        RS
      BBBB=X(I)-XIO-HTS2DO(I)                                           RS
      IF(BBBB.LE.0.0) GO TO 70                                          RS
      HTS2K(I)=0.0                                                      RS
      HHTS2(I)=1                                                        RS
   70 CONTINUE                                                          RS
      HTS2DU(I)=(-HTS2U(I)/FHTS2K(I))+HTS2DO(I)                         RS
      CCCC=X(I)-XIO-HTS2DU(I)                                           RS
      IF(CCCC.GE.0.0) GO TO 80                                          RS
      HTS2K(I)=0.0                                                      RS
      HHTS2(I)=4                                                        RS
   80 CONTINUE                                                          RS
C                                                                       RS
C     FORM TOTAL STIFFNESS MATRIX                                       RS
C                                                                       RS
      DO 4 L=1,NM                                                       RS
      DO 4 J=1,NM                                                       RS
    4 TK(L,J)=FK(L,J)                                                   RS
      TK(1,1)=TK(1,1)+AMS1K(1)+AMS2K(1)+HTS1K(1)+HTS2K(1)               RS
      IF(NM.EQ.1) GO TO 5                                               RS
      DO 6 L=2,NM                                                       RS
      AK=AMS1K(L)+AMS2K(L)+HTS1K(L)+HTS2K(L)                            RS
      TK(L-1,L-1)=TK(L-1,L-1)+AK                                        RS
      TK(L-1,L)=TK(L-1,L)-AK                                            RS
```

LIST 4 (continued)

```
      TK(L,L-1)=TK(L,L-1)-AK                                          RS
    6 TK(L,L)=TK(L,L)+AK                                              RS
    5 CONTINUE                                                        RS
C                                                                     RS
C     INTEGRATION PROCESS                                             RS
C                                                                     RS
C     SET UP VECTOR A OF INTEGRATION PROCESS                          RS
      DO 500 I=1,NM                                                   RS
      AA(I)=-6.0*VEL(I)/DT-3.0*ACC(I)                                 RS
C                                                                     RS
C     SET UP VECTOR B OF INTEGRATION PROCESS                          RS
  500 BB(I)=-3.0*VEL(I)-0.5*DT*ACC(I)                                 RS
C                                                                     RS
C     SET UP VECTOR DELTA R OF INTEGRATION PROCESS                    RS
      CALL ACCEL(DXEDD)                                               RS
      DO 503 L=1,NM                                                   RS
      CB=0                                                            RS
      DO 502 J=1,NM                                                   RS
  502 CB=CB+DM(L,J)*BB(J)                                             RS
  503 DR(L)=-ZM(L)*DXEDD-ZM(L)*AA(L)-CB                               RS
      IF(TIME.EQ.0.) GO TO 511                                        RS
C     QUERY WHETHER STIFFNESS HISTORY INDICATOR HAS CHANGED           RS
      DO 504 I=1,NM                                                   RS
  504 IF(HTF(I)-PHTF(I).NE.0) GO TO 505                               RS
      GO TO 521                                                       RS
  505 DO 506 I=1,NM                                                   RS
  506 PHTF(I)=HTF(I)                                                  RS
C     SET UP MATRIX K STAR                                            RS
  511 DO 514 I=1,NM                                                   RS
      DO 512 J=1,NM                                                   RS
  512 SK(I,J)=3.0*DM(I,J)/DT+TK(I,J)                                  RS
  514 SK(I,I)=6.0*ZM(I)/DT**2+SK(I,I)                                 RS
C                                                                     RS
C     INVERT MATRIX K STAR SINCE CHANGED FROM PREVIOUS VALUE          RS
      DO 521 N=1,NM                                                   RS
      D=SK(N,N)                                                       RS
      IF(D.NE.0.0) GO TO 517                                          RS
      WRITE(6,516)                                                    RS
  516 FORMAT(1X,15HSINGULAR MATRIX////)                               RS
      STOP                                                            RS
  517 DO 518 J=1,NM                                                   RS
  518 SK(N,J)=-SK(N,J)/D                                              RS
      DO 520 I=1,NM                                                   RS
      IF(N-I.EQ.0) GO TO 520                                          RS
      DO 519 J=1,NM                                                   RS
      IF(N-J.EQ.0) GO TO 519                                          RS
      SK(I,J)=SK(I,J)+SK(I,N)*SK(N,J)                                 RS
  519 CONTINUE                                                        RS
  520 SK(I,N)=SK(I,N)/D                                               RS
      SK(N,N)=1.0/D                                                   RS
  521 CONTINUE                                                        RS
      DO 522 I=1,NM                                                   RS
      AX(I)=0.0                                                       RS
      DO 524 J=1,NM                                                   RS
  524 AX(I)=AX(I)+SK(I,J)*DR(J)                                       RS
C     COMPUTE MASS DISPLACEMENT                                       RS
  522 X(I)=X(I)+AX(I)                                                 RS
      DO523 I=1,NM                                                    RS
      IF(ABS(X(I)).LE.ABS(BX(I)))GO TO 523                            RS
      TX(I)=TIME                                                      RS
      BX(I)=X(I)                                                      RS
  523 CONTINUE                                                        RS
C                                                                     RS
C     SET UP VELOCITY AND ACCELERATION VECTORS FOR NEXT CYCLE         RS
  525 DO 526 I=1,NM                                                   RS
      VEL(I)=VEL(I)+3.0*AX(I)/DT+BB(I)                                RS
  526 ACC(I)=ACC(I)+6.0 * AX(I)/(DT*DT)+AA(I)                         RS
      IF(TIME.GE.DT) GO TO 560                                        RS
      WRITE(6,555)                                                    RS
  555 FORMAT('0',23X,50('*')//'/    TIME',20X,'DISPLACEMENT OF MASSES',2RS
     10X,'HISTORY RECORD'/)                                           RS
  560 TIME=TIME+DT                                                    RS
      KC=KC+1                                                         RS
      IF(NPS-KC.GT.0) GO TO 591                                       RS
      IF(NPFLAG.NE.0) GO TO 564                                       RS
      NXP=NXP+1                                                       RS
      IF(NXP.GT.1000) STOP 4                                          RS
      XP1(NXP)=X(1)                                                   RS
      XP2(NXP)=X(2)                                                   RS
      XP3(NXP)=X(3)                                                   RS
  564 NPFLAG=1-NPFLAG                                                 RS
      DO 565 I=1,NM                                                   RS
  565 HTF(I)=1000*HMS1(I)+100*HMS2(I)+10*HHTS1(I)+HHTS2(I)            RS
      IF(NWRTE.EQ.0) GO TO 590                                        RS
      WRITE(6,570) TIME,(X(I),I=1,NM)                                 RS
  570 FORMAT(1X,10E13.4)                                              RS
      WRITE(6,581)(HTF(I),I=1,NM)                                     RS
  581 FORMAT(1H+,60X,7I10/)                                           RS
  590 KC=0                                                            RS
  591 IF(TIME.LT.TR) GO TO 240                                        RS
      TIME=TIME+DT                                                    RS
  592 WRITE(6,593)                                                    RS
  593 FORMAT('0',34X,14HMAXIMUM VALUES/)                              RS
      WRITE(6,594) (TX(I),BX(I),I=1,NM)                               RS
  594 FORMAT(30X,2E13.4)                                              RS
      WRITE(6,597)                                                    RS
  597 FORMAT('0',24X,37HTOTAL YIELD DISPLACEMENT OF MS BRACES/)       RS
      WRITE(6,598)(TADJ1(I),I=1,NM),(TADJ2(I),I=1,NM)                 RS
  598 FORMAT(25X,3E14.6)                                              RS
  599 IF(NPLOT.EQ.0) GO TO 600                                        RS
      MYTICK=40                                                       RS
      IYTICK=15                                                       RS
      LOGICAL*1 ILAB1(2)/'$2'/,ILAB2(4)/'TIME'/,ILAB3(9)/'(SECONDS)'/,RS
     1ILAB4(21)/'DISPLACEMENT (INCHES)'/                              RS
      IY25=-15                                                        RS
      IF(XP3(NXP).LT.0.0) IY25=5                                      RS
      CALL AINIT(1200)                                                RS
      CALL AGRID(200,400,25,1,32,0,1,1)                               RS
      CALL AGRID(200,100,1,MYTICK,0,IYTICK,1,1)                       RS
      CALL ASCA(140,95,0,15,-20,1,41,1,2)                             RS
      CALL ACRIG(200,400)                                             RS
      CALL ALAB(795,-5,ILAB1,2,1,2)                                   RS
      CALL ASCA(760,IY25,0,0,25,0,1,1,2)                              RS
      CALL ALAB(810,-30,ILAB2,4,1,2)                                  RS
      CALL ALAB(810,-45,ILAB3,9,1,2)                                  RS
      CALL ALAB(-50,-105,ILAB4,21,1,4)                                RS
      CALL ALINEX(0,1,XP1,NXP,0.0,6.66667)                            RS
      CALL ALINEX(0,1,XP2,NXP,0.0,6.66667)                            RS
      CALL ALINEX(0,1,XP3,NXP,0.0,6.66667)                            RS
      CALL AEND                                                       RS
```

LIST4 (continued)

```
600 RETURN                                                              RS
    END                                                                 RS
```

```
    SUBROUTINE ACCEL(DXEDD)                                             RS
    COMMON DT                                                           RS
    DIMENSION DXGDD(8000)                                               RS
    K=K+1                                                               RS
    IF(K.GT.1) GO TO 1                                                  RS
    CALL RDBERG(DXGDD)                                                  RS
    DXEDD=DXGDD(K)                                                      RS
    RETURN                                                              RS
    END                                                                 RS
```

```
    SUBROUTINE RDBERG(DXGDD)                                            RS
    COMMON DT                                                           RS
    DIMENSION A(700),T(700),DXGDD(8000),AEQ(20)                         RS
    IF(DT.EQ.0.) DT=1.0/256.0                                           RS
    READ(5,400) NCARD,SF                                                RS
400 FORMAT(I5,G10.0)                                                    RS
    IF(NCARD.EQ.0)  NCARD=20                                            RS
    IF(SF.EQ.0.) SF=1.0                                                 RS
    READ(5,410) AEQ                                                     RS
410 FORMAT(20A4)                                                        RS
420 FORMAT('0',30X,'DETAILS OF EARTHQUAKE BEING USED...'//              RS
   133X,20A4///                                                         RS
   233X,37HNUMBER OF EARTHQUAKE CARDS TO BE READ,13X,1H=,I10//          RS
   333X,20H1G E/Q RECORD FACTOR,30X,1H=,F10.2//                         RS
   433X,18HTIME STEP INTERVAL,32X,1H=,F10.8//)                          RS
    WRITE(6,420) AEQ,NCARD,SF,DT                                        RS
    NPOINT=4*NCARD                                                      RS
425 NX=1                                                                RS
    DO 430 I=1,NPOINT,4                                                 RS
    READ(5,428) NC,(T(I+K-1),A(I+K-1),K=1,4),NR                         RS
428 FORMAT(I3,4(F8.4,F9.6),I4)                                          RS
    IF(NC.NE.NX) GO TO 463                                              RS
430 NX=NC+1                                                             RS
    TNPT=T(NPOINT)                                                      RS
    AXGDD=0.0                                                           RS
    PXGDD=0.0                                                           RS
    J=1                                                                 RS
    I=0                                                                 RS
    TIME=0                                                              RS
435 AXGDD=SF*386.4*((TIME-T(J))*(A(J)-A(J+1))/(T(J)-T(J+1))+A(J))       RS
    ADXGDD=AXGDD-PXGDD                                                  RS
    PXGDD=AXGDD                                                         RS
    I=I+1                                                               RS
    DXGDD(I)=ADXGDD                                                     RS
438 TIME=TIME+DT                                                        RS
    IF(I.EQ.8001) STOP2                                                 RS
    IF(TIME-T(J+1).LE.0.) GO TO 435                                     RS
450 J=J+1                                                               RS
    IF(J.EQ.701) STOP3                                                  RS
    IF(T(J).EQ.T(J+1)) GO TO 450                                        RS
    IF((J-NPOINT).LE.0.) GO TO 435                                      RS
460 NP=I                                                                RS
    GO TO 470                                                           RS
463 WRITE(6,465) NC                                                     RS
465 FORMAT('0CARD NO.'I4'OUT OF ORDER')                                 RS
    PAUSE 'CORRECT DATA CARD ORDER, RESTART AT NO 1'                    RS
    GO TO 425                                                           RS
470 RETURN                                                              RS
    END                                                                 RS
```

LIST4 (continued)

INPUT DATA ECHO CHECK

MASS MATRIX

0.1369E 05 0.1294E 05 0.6780E 06 0.2480E 06

UNBRACED FRAME LATERAL STIFFNESS MATRIX

0.134000E 05
-.131000E 05 0.451000E 04
-.104000E 01 -.131000E 05 0.221000E 05
-.127000E 05 0.655000E 01 0.444000E 04
-.127000E 05 0.992000E 04 -.257000E 03 0.559000E 02
-.489000E 02 -.238000E 03 0.235000E 03

DAMPING MATRIX

0.516396D 03
-.504481D 03 0.170984D 03
0.215271D 01 -.504481D 03 0.851413D 03
-.489077D 03 -.188314D 01 0.173680D 03
-.489077D 03 0.399969D 03 -.216538D 01 -.400504D-01 0.252240D 00
-.989707D 01 0.156157D 02

BRACING DETAILS

4.82	566.	0.0	0.288E 06	0.110E 05	49.7	0.312E 08	
4.82	566.	0.0	0.298E 06	0.110E 05	49.7	0.312E 08	
.786	566.	0.0	0.0		49.7	0.289E 08	
.786	566.	0.0	0.0		49.7	0.289E 08	

BRACING DETAILS

7.10	530.	0.0	0.425E 06	0.160E 05	52.3	0.312E 08	
7.10	530.	0.0	0.425E 06	0.160E 05	52.3	0.312E 08	
.786	530.	0.0	0.0		52.3	0.289E 08	
.786	530.	0.0	0.0		52.3	0.289E 08	

BRACING DETAILS

4.66	509.	0.0	0.518E 06	0.190E 05	53.4	0.312E 08	
4.66	509.	0.0	0.518E 06	0.190E 05	53.4	0.312E 08	
.786	509.	0.0	0.0		53.4	0.289E 08	
.786	509.	0.0	0.0		53.4	0.289E 08	

BRACING DETAILS

4.82	566.	0.100E 06	0.288E 06	0.100E 05	49.7	0.312E-08	
4.82	566.	0.100E 06	0.288E 06	0.100E 05	49.7	0.312E-08	
.786	566.	0.150E 06	0.778E 06	49.7		0.289E-08	
.786	566.	0.150E 06	0.778E 06	49.7		0.289E-08	

DETAILS OF EARTHQUAKE BEING USED...

EARTHQUAKE BEING USED IS THE EL CENTO MAY 1940 ONE.(NORTH-SOUTH)...

NUMBER OF EARTHQUAKE CARDS TO BE READ = 125

1G E/Q RECORD FACTOR = 1.CC

TIME STEP INTERVAL =0.CC39C625

****

TIME	DISPLACEMENT OF MASSES				HISTORY RECORD		
0.1563E-01	-0.3398E-03	-0.3739E-03	-0.3700E-03	-0.3700E-03	1214	1214	2141
0.3125E-01	-0.1149E-02	-0.1469E-02	-0.1491E-02	-0.1492E-02	1214	1214	1244
0.4688E-01	-0.1825E-02	-0.2686E-02	-0.3027E-02	-0.3030E-02	1214	1214	1244
0.6250E-01	-0.2268E-02	-0.3756E-02	-0.4786E-02	-0.4803E-02	1214	1214	1244
0.7813E-01	-0.3056E-02	-0.5246E-02	-0.7047E-02	-0.7109E-02	1214	1214	1244
0.9375E-01	-0.4687E-02	-0.7801E-02	-0.1018E-01	-0.1034E-01	1214	1214	1244
0.1094E 00	-0.7237E-02	-0.1158E-01	-0.1455E-01	-0.1487E-01	1214	1214	1244
0.1250E 00	-0.1008E-01	-0.1610E-01	-0.2012E-01	-0.2069E-01	1214	1214	1244
0.1406E 00	-0.1254E-01	-0.2069E-01	-0.2650E-01	-0.2743E-01	1214	1214	1244
0.1563E 00	-0.1466E-01	-0.2517E-01	-0.3326E-01	-0.3473E-01	1214	1214	1244
0.1719E 00	-0.1693E-01	-0.2972E-01	-0.4008E-01	-0.4232E-01	1214	1214	1244
0.1430E 02	0.3837E 01	0.4916E 01	0.5598E 01	-0.2324E 01	3144	2144	2144
0.1431E 02	0.3768E 01	0.5083E 01	0.6085E 01	-0.2310E 01	2144	2144	2144
0.1433E 02	0.3735E 01	0.5258E 01	0.6560E 01	-0.2297E 01	2144	2144	2144
0.1434E 02	0.3949E 01	0.5586E 01	0.6959E 01	-0.2284E 01	3144	2144	2144
0.1436E 02	0.4397E 01	0.6055E 01	0.7340E 01	-0.2269E 01	3144	2144	2144
0.1437E 02	0.4598E 01	0.6552E 01	0.7688E 01	-0.2250E 01	3144	2144	2144
0.1439E 02	0.5573E 01	0.7010E 01	0.8067E 01	-0.2228E 01	3144	2144	2144
0.1441E 02	0.5955E 01	0.7368E 01	0.8299E 01	-0.2203E 01	3144	2144	2144
0.1442E 02	0.6C99E 01	0.7570E 01	0.8564E 01	-0.2176E 01	3144	2144	2144
0.1444E 02	0.6C65E 01	0.7643E 01	0.8798E 01	-0.2150E 01	2144	2144	2144
0.1445E 02	0.5989E 01	0.7682E 01	0.8998E 01	-0.2124E 01	2144	2144	2144
0.1447E 02	0.6C27E 01	0.7775E 01	0.9158E 01	-0.2099E 01	2144	2144	2144
0.1448E 02	0.6198E 01	0.7944E 01	0.9279E 01	-0.2073E 01	3144	2144	2144
0.1450E 02	0.6398E 01	0.8124E 01	0.9362E 01	-0.2047E 01	3144	2144	2144
0.1452E 02	0.6555E 01	0.8223E 01	0.9411E 01	-0.2018E 01	3144	2144	2144
0.1453E 02	0.6597E 01	0.8241E 01	0.9425E 01	-0.1989E 01	3144	2144	2144
0.1455E 02	0.6511E 01	0.8189E 01	0.9405E 01	-0.1960E 01	2144	2144	2144
0.1456E 02	0.6362E 01	0.8071E 01	0.9369E 01	-0.1931E 01	2144	2144	2144
0.1458E 02	0.6267E 01	0.7943E 01	0.9252E 01	-0.1904E 01	2144	2144	2144
0.1459E 02	0.6254E 01	0.7869E 01	0.9113E 01	-0.1883E 01	2144	2144	2144
0.1461E 02	0.6270E 01	0.7814E 01	0.8931E 01	-0.1868E 01	2144	2144	2144
0.1462E 02	0.6231E 01	0.7695E 01	0.8711E 01	-0.1859E 01	2144	2144	2144
0.1464E 02	0.6084E 01	0.7466E 01	0.8454E 01	-0.1857E 01	2144	2144	2144
0.1465E 02	0.5863E 01	0.7180E 01	0.8162E 01	-0.1860E 01	2144	2144	2144
0.1467E 02	0.5667E 01	0.6897E 01	0.7842E 01	-0.1863E 01	2144	2144	2144
0.1469E 02	0.5547E 01	0.6656E 01	0.7495E 01	-0.1865E 01	2144	2144	2144
0.1470E 02	0.5466E 01	0.6440E 01	0.7126E 01	-0.1864E 01	2144	2144	2144
0.1472E 02	0.5369E 01	0.6191E 01	0.6741E 01	-0.1860E 01	2144	2144	2144
0.1473E 02	0.5152E 01	0.5875E 01	0.6345E 01	-0.1850E 01	2144	2144	2144
0.1475E 02	0.4800E 01	0.5513E 01	0.5938E 01	-0.1838E 01	2144	2144	2144
0.2481E 02	-0.2196E 01	-0.1943E 01	-0.1642E 01	-0.1494E 01	1244	2144	2144
0.2483E 02	-0.2263E 01	-0.2061E 01	-0.1853E 01	-0.1772E 01	1244	2144	2144
0.2484E 02	-0.2332E 01	-0.2121E 01	-0.2013E 01	-0.1943E 01	1244	2144	2144
0.2486E 02	-0.2404E 01	-0.2352E 01	-0.2172E 01	-0.2109E 01	1244	2144	2144
0.2487E 02	-0.2474E 01	-0.2422E 01	-0.2328E 01	-0.2266E 01	1244	2144	2144
0.2489E 02	-0.2547E 01	-0.2541E 01	-0.2484E 01	-0.2423E 01	1244	2144	2144
0.2491E 02	-0.2616E 01	-0.2659E 01	-0.2640E 01	-0.2577E 01	1244	2144	2144
0.2492E 02	-0.2684E 01	-0.2776E 01	-0.2796E 01	-0.2732E 01	1244	2144	2144
0.2494E 02	-0.2753E 01	-0.2894E 01	-0.2944E 01	-0.2889E 01	1244	2144	2144
0.2495E 02	-0.2823E 01	-0.3014E 01	-0.3112E 01	-0.3046E 01	1244	2144	2144
0.2497E 02	-0.2893E 01	-0.3133E 01	-0.3269E 01	-0.3203E 01	1244	2144	2144
0.2499E 02	-0.2964E 01	-0.3252E 01	-0.3426E 01	-0.3370E 01	1244	2144	2144
0.2500E 02	-0.3032E 01	-0.3369E 01	-0.3581E 01	-0.3537E 01	1244	2144	2144

MAXIMUM VALUES

0.1452E 02 0.6600E 01
0.1452E 02 0.8243E 01
0.1453E 02 0.9425E 01
0.2562E 01 0.1029E 02

TOTAL YIELD DISPLACEMENT OF MS PRACES

0.4CC511E 01 0.0 0.0
0.0 -0.143373E 01 0.0
0.0 0.0

STOP 0

LIST 4 OUTPUT

the appropriate insertion on the control card at the start of the program. Typical line printer output is shown at the end of LIST 4 and the results of using the punched output to activate the X-Y plotter on line to the University of Canterbury's IBM 1620 computer are shown in the displacement/time plots presented in Chapter Six.

The subroutine RDBERG is used to produce a set of exciting accelerations at selected times for use in the numerical integration section of XBFRME, by suitable modification of Berg type[70] digitised accelerograms. In their original state these accelerograms exist in digitised form with four successive peak values of acceleration, together with the time of each event, listed on each card. It is assumed that the accelerograms are fully defined by these points, a series of straight lines drawn between the points being sufficient to complete the traces. For the purpose of numerical integration it is necessary to convert the Berg digitised records to a set of acceleration values at equal (preselected) time intervals. The principle used in RDBERG is that of establishing the gradient of the accelerogram between two consecutive time points and then determining the value of acceleration at the time interval chosen by calculating the product of the gradient and the time elapsed since the last peak on the accelerogram. (See Figure 5.2). The procedure adopted has the effect of cutting off the peaks of the original accelerogram in all cases other than when the exact multiple of the selected integration time step coincides with the time at which the peak occurs, but this is considered to be satisfactory providing the step interval is small enough to reduce the neglected peaks to an insignificant proportion

of the whole accelerogram.

V.6 (iv) Input data XBFRME program

1. First Card, NM, NPS, TR, DT, CG, CR, NPLOT, NWRTE

Format (2I5, 4G 10.0, 2I5)

N	=	Number of Masses (one per storey)
NPS	=	Number of time steps before printing
TR	=	Length of earthquake record (seconds)
DT	=	Step interval (seconds)
CG	=	Damping Coefficient
CR	=	Damping Coefficient
NPLOT	=	0 for no plotter output, 1 for plotter output
NWRTE	=	0 for no printed output, 1 for printed output

2. Mass Card(s). (Actually in lb. weight units) From bottom up.

Format (8E10.4)

3. Unbraced Frame Lateral Stiffness Matrix Card(s).

Format (8G10.3)

4. Bracing Detail Cards (4 Cards necessary for each bay).

For each bay, Two Mild Steel Cards (Format 8G10.3)

Containing: (N.B. ? represents digit 1 or digit 2).

A	=	Bracing Rod Area (in^2)
ZL	=	Bracing Rod Length (in)
AMS?P	=	Prestressing Force in Bracing Rod (lbs)
AMS?Y	=	Axial Force at which Bracing Rod will Yield (lbs)
MS?PYK	=	Post yield axial stiffness of Bracing Rod (lb/in)
ANG	=	Angle to horizontal of Bracing Rod (degrees)
EMS	=	Elastic Modulus (lb/in^2)

and, for each bay, a second pair of cards, for the high tensile steel containing:

A	=	Bracing Rod Area (in^2)
ZL	=	Bracing Rod Length (in)
HTS?P	=	Prestressing Force in Bracing Rod (lbs)
HTS?U	=	Axial Force at which Bracing Rod will break (lbs)
ANG	=	Angle to horizontal of Bracing Rod (degrees)
EHT	=	Elastic Modulus (lbs/in^2)

(N.B. for a three storey frame there will be a total of twelve bracing cards, read in storey by storey, MS1, MS2, HTS1, HTS2 from the ground up).

5. A Control Card NCARD, SF

Format (I5, G10.0)

NCARD = Number of Earthquake Record Cards to be read (consistent with TR in first card).
SF = Scale factor by which earthquake accelerations may be multiplied.

6. Digitised Earthquake Record Cards.

First one Title Card - Format (20A4)

Remainder, in numerical sequence up to NCARD,

Berg type Format I3, 4 (F8.4, F9.6), I4

ISC = Sequence Number
T(I), G(I), I-2, 5 = four co-ordinate points - time (seconds), acceleration (fraction of g) for each point
NR = Earthquake Record Number

OUTPUT DATA XBFRME PROGRAM

1. Input Data Echo Check - Unbraced Frame Lateral Stiffness Matrix - Computed Damping Matrix - Input Bracing Details - Earthquake Record Details.

2. Written Output - Time, Displacement of Masses and Bracing History Record.

3. Plotter Output - Time and Mass Displacement values for Plotting on off-line X-Y plotter.

V.6 (v) Symbol table XBFRME program

A(main program)	= Bracing Cross Section Area
A(subroutine)	= Ground Acceleration, (proportion of gravity)
AA	= Vector used in the integration process
ACC	= Vector used in the integration process
ANG	- Angle of Bracing rod to horizontal
ADJ1	= Adjustment to Bracing Rod 1 elastic range after yield
ADJ2	= Adjustment to Bracing Rod 2 elastic range after yield
AK	= Incremental Lateral Stiffness due to Bracing
AMS1DN	= Position of zero load in M.S. Bracing Rod 1
AMS1DO	= Original zero load position for M.S. Bracing Rod 1
AMS1DY	= Position at which M.S. Bracing Rod 1 just yields
AMS1K	= Axial Stiffness of M.S. Bracing Rod 1
AMS1P	= Prestressing force in M.S. Bracing Rod 1
AMS1Y	= Elastic limit force in M.S. Bracing Rod 1
AMS2DN	= Position of zero load in M.S. Bracing Rod 2
AMS2DO	= Original zero load position for M.S. Bracing Rod 2
AMS2DY	= Position at which M.S. Bracing Rod 2 just yields
AMS2K	= Axial Stiffness of M.S. Bracing Rod 2
AMS2P	= Prestressing Force in M.S. Bracing Rod 2
AX	- Incremental deflection
BB	= Vector used in the integration process
BC	= Vector used in the integration process
BX	= Maximum displacement
CG	- Damping Coefficient
CR	= Damping Coefficient
DXEDD	= Exciting acceleration
DXGDD (subroutine)	= Exciting acceleration
DM	= Damping Matrix
DR	= Vector used in the integration process
DT	- Time step
EHT	= Elastic Modulus H.T.S. Bracing
EMS	= Elastic Modulus M.S. Bracing
FHTS1K	= Initial axial stiffness of H.T.S. Bracing Rod 1
FHTS2K	= Initial axial stiffness of H.T.S. Bracing Rod 2
FK	= Unbraced frame lateral stiffness matrix
FMS1K	= Initial axial stiffness of M.S. Bracing Rod 1
FMS2K	= Initial axial stiffness of M.S. Bracing Rod 2
HTF	= Bracing Rod load state record
HHTS1	= History indicator H.T.S. Bracing Rod 1
HHTS2	= History indicator H.T.S. Bracing Rod 2
HMS1	= History indicator M.S. Bracing Rod 1
HMS2	= History indicator M.S. Bracing Rod 2
HTS1DO	= Original zero load position for H.T.S. Bracing Rod 1
HTS1DU	= Ultimate load position for H.T.S. Bracing Rod 1
HTS1K	= Axial Stiffness of H.T.S. Bracing Rod 1
HTS1P	= Prestressing force in H.T.S. Bracing Rod 1
HTS1U	= Ultimate load in H.T.S. Bracing Rod 1
HTS2DO	= Original zero load position for H.T.S. Bracing Rod 2

HTS2DU	- Ultimate load position for H.T.S. Bracing Rod 2
HTS2K	- Axial Stiffness of H.T.S. Bracing Rod 2
HTS2P	- Prestressing force in H.T.S. Bracing Rod 2
HTS2U	- Ultimate load in H.T.S. Bracing Rod 2
MS1PYK	- Post yield axial stiffness of M.S. Bracing Rod 1
MS2PYK	- Post yield axial stiffness of M.S. Bracing Rod 2
NM	- Number of masses in system
NPLOT	- Punched Card output control
NPS	- Number of time steps per write step
NWRTE	- Line Printer output control
PHTF	- Bracing Rod load state record in previous cycle
PX	- Deflection in previous cycle
SK	- Vector used in integration process
TADJ1	- Total yield displacement of M.S. Bracing Rod 1
TADJ2	- Total yield displacement of M.S. Bracing Rod 2
T (subroutine)	- Time
TR	- Total length of accelerogram record
TX	- Time at which maximum deflection occurs
VEL	- Vector used in the integration process
W	- Weight
X	- Displacement
XP1	- Displacement arrays used in Plotter output
XP2	- Displacement arrays used in Plotter output
XP3	- Displacement arrays used in Plotter output
ZM	- Mass vector
ZL	- Length of Bracing Rods

V.6 (vi) Checks on XBFRME program

Because of the novelty of the XBFRME analysis it was not possible, in the case of multiphase bracing response, to check it out against results obtained from a completely independent procedure. However it was confirmed that the integration, RDBERG and output routines do function correctly. This was done both by reading in bracing details which resulted in the frame being effectively unbraced (readily achieved by setting the bracing elastic modulus at 30×10^{-6} instead of $30 \times 10^{+6}$) and subsequently by selecting a sufficiently high level of yield load in the braces (10^{8} lbs for the frame analysed in Chapter Six) to ensure that the bracing always responded in the extended elastic range throughout the full analysis.

Independent analyses were made of the resulting two sets of fully elastic systems using the library program ELRES (LIST LP4) which, as stated earlier, is based on a numerical integration solution of equation 4(1). Virtually identical displacement/time plots to those obtained from ELRES were determined using XBFRME providing that the time step interval was set to be sufficiently small. Evidence of non-agreement was obtained at XBFRME time step intervals down to less than 1/100 second but no difference was evident when a time step of 1/256 second was used. This result is entirely consistent with that of an earlier investigation[63] of the use of the IBM 360/44 for this type of numerical analysis and consequently all subsequent analyses using XBFRME were carried out with the integration time step set at 1/256 second. This also ensured that the displacement step interval would be small enough to make negligible the error introduced by projecting across the step interval at a gradient defined by the last value of stiffness (referred to in section V.6 (ii) above).

Comparisons between ELRES and XBFRME were made for both undamped and damped elastic systems. Virtually identical results were obtained from the two analysis procedures, as is illustrated by the results for the elevated water tank tower discussed in Chapter Six.

Since verification that KLAT functions satisfactorily had been previously made (section V.6 (ii) above) it was concluded that no other validity checks on XBFRME could be made with advantage.

V.7 Summary of the Use of Digital Computer Programs in the Determination of the Seismic Response of Elevated Water Tanks

Subsequent to the idealisation of the supporting structure to a discrete mass system with specified geometric and mass characteristics, the following steps are followed to determine the seismic response of an elevated water tank supported on a cross braced frame tower.

A. The elastic stiffness characteristics of the unbraced tower framework are computed. (Program '2-D STRUCTURE', LIST LP1, Appendix IV).

B. The post yield characteristics of the mild steel bracing members are calculated (Program 'TRUEFORCE', Chapter Three).

C. The convective effect of the water in the tank (i.e. the two mass representation of the tank and contents) is determined. (Program 'MODANAL', Chapter Two).

D. Using the results of steps A and C, the normal mode frequencies of lateral vibration of the undamped tower supporting the two mass representation of the tank and contents is computed. (Program 'NORMOD', LIST LP2, Appendix IV).

E. Using the normal mode frequencies determined in step D, the damping characteristics are established. (Program 'DAMPING MATRIX', LIST LP2A, Appendix IV).

F. Using the results of steps A, B, C and E the seismic response of the elevated water tower is determined. (Program 'XBFRME', Chapter Five, Section 6).

C H A P T E R S I X

SEISMIC RESPONSE ANALYSES

VI.1 Introduction

To illustrate the analysis procedures presented in previous sections of this thesis the water tank described by Moran and Cheney[21] was used as the basis of a prototype system and the analyses carried out on this structure are described below.

VI.2 The Structure Examined

The structure used for the verification and sensitivity analyses presented in this chapter is shown diagrammatically in Figure 6.1. The tower has welded tubular columns and circular section bracing rods. The columns are 22 in. inside diameter and of 5/16 in. thickness in the upper section, of 3/8 in. thickness in the centre section, and of 7/16 in. thickness in the lower section. The bracing rods are of $2\frac{3}{8}$ in. diameter for the upper section, $2\frac{1}{8}$ in. diameter for the centre section and $1\frac{3}{4}$ in. for the lower section. The horizontal strut members are each fabricated from two channels (see the detail on Figure 6.1). Between the centre and upper sections the channel with the horizontal web is of 8 in. web dimension and weighs 11.5 lb/ft. whereas the channel having horizontal flanges is of 9 in. web dimensions and weighs 13.4 lb/ft. Between the lower and centre sections the channel

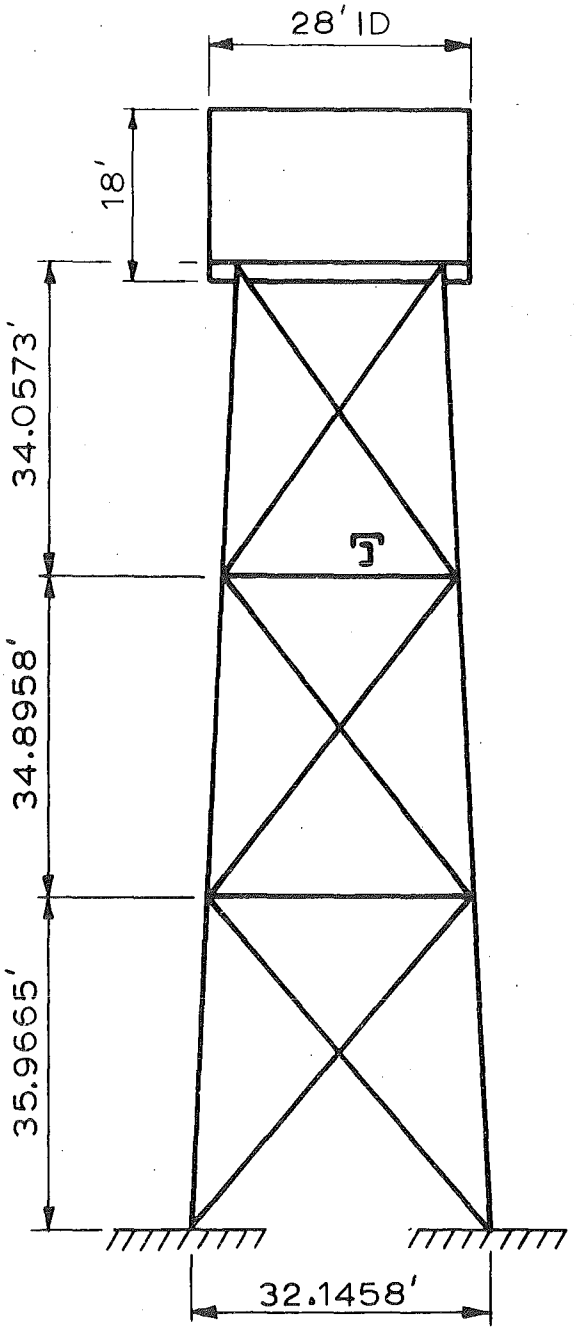

Fig.6.1 : Elevated Water Tank Analysed
(After Moran & Cheney)

with the horizontal web is of 8 in. web size and weighs 11.5 lb/ft. whereas the channel with the horizontal flanges is of 10 in. web dimension and weighs 15.3 lb/ft. Moran and Cheney give the weight of the tank and its contents as 926,000 lb.

For the purpose of this study a cylindrical tank is used in place of the original one which had a domed top and bottom.

A summary of the calculations undertaken in modelling the three storey tower frame is presented in Appendix V. Since the capacity of the tank is given as 100,000 (U.S.) gallons when full the contents were assumed to weigh 832,000 lbs hence the tank structure weight was deduced to be 94,000 lbs.

The self-weight mass distribution when the tank is empty, is

13,688 lbs M_1 (See Figure 4.2)
12,942 lbs M_2
99,110 lbs $M_3 + m_s$

and these values are used in the following analyses. (Note the computer programs accept weights rather than masses and factor accordingly in the computational procedure).

VI.3 Digital Computer Analyses With No Convective Action of the Water

(Note: All numerical values in lb.in. units)

Step A. The unbraced frame flexibility matrix for the three mass system may be determined using the '2-D STRUCTURE' Program and has form:

$$10^{-4} \times \begin{bmatrix} 5.642 & 7.513 & 7.236 \\ 7.513 & 11.86 & 12.07 \\ 7.236 & 12.07 & 13.50 \end{bmatrix}$$

This, when inverted, using the matrix inversion program (LIST, LP3), gives the lateral stiffness matrix for the unbraced frame which has form

$$10^{4} \times \begin{bmatrix} 1.34 & -1.31 & 0.453 \\ -1.31 & 2.22 & -1.28 \\ 0.453 & -1.28 & 0.975 \end{bmatrix}$$

Step B. The post yield axial stiffness characteristics of the bracing rods, determined using program 'TRUEFORCE', are presented in Figure 6.2, from these results values of 5,500 lb/in., 8,000 lb/in. and 9,500 lb/in. may be deduced for the effective post yield stiffnesses of each bracing rod in the lower, middle and upper braced bays.

Step D. When the mass and stiffness properties previously established are used as input to the normal mode analysis program, natural frequencies of 0.2716 cps and 5.774 cps may be determined for the first and second modes of the unbraced tower supporting an empty tank.

Step E. Since it is generally accepted that elevated water tank structures exhibit relatively low percentage critical damping characteristics, 2% critical was selected for each

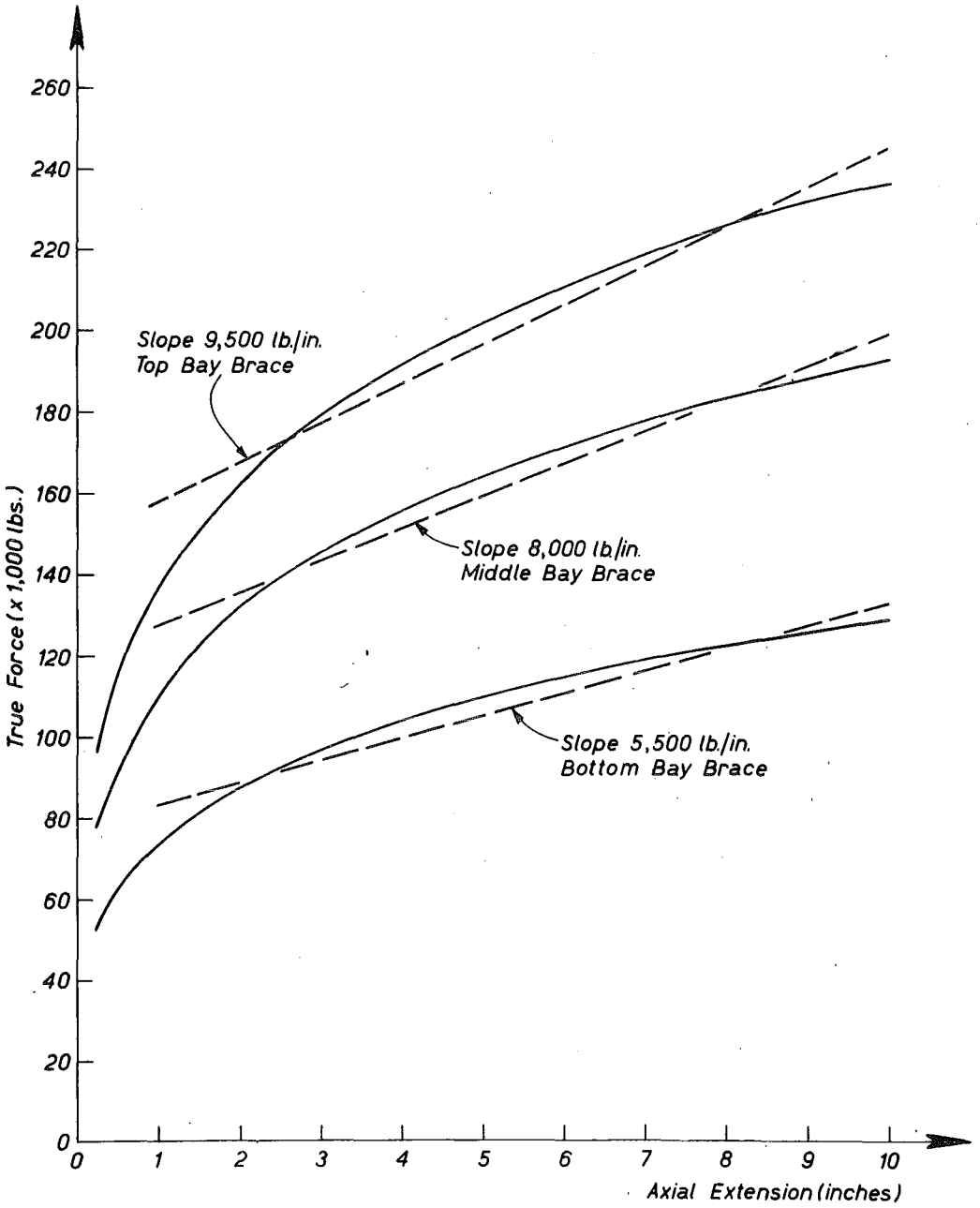

Fig. 6.2 : TRUE FORCE / TRUE DEFLECTION CURVES FOR BRACING RODS.

of the first two modes of vibration and, when used as input to the library program 'DAMPING MATRIX', damping coefficients of CG = 0.00105 and CR = 0.0652 may be determined, also the damping matrix

$$10^2 \times \begin{bmatrix} 0.164 & -0.138 & 0.048 \\ -0.138 & 0.256 & -0.135 \\ 0.048 & -0.135 & 0.270 \end{bmatrix}$$

may be derived.

Step F. The input data determined as described above was used in the program XBFRME together with the digitised North/South accelerations of the 1940 El Centro earthquake, to determine a series of displacement/time histories as shown in the accompanying figures.

The displacement response of each of the three masses of the system to the El Centro accelerogram factored by 1.0, when no damping is allowed for in the unbraced frame supporting an empty tank, is shown in Figure 6.3. The result of this essentially elastic analysis is identical to that obtained by the use of the library program ELRES to undertake the same response calculation.

Maximum displacements were determined as follows

Mass 1 : 10.73 ins at 5.449 secs
Mass 2 : 15.99 ins at 5.402 secs
Mass 3 : 17.87 ins at 5.250 secs

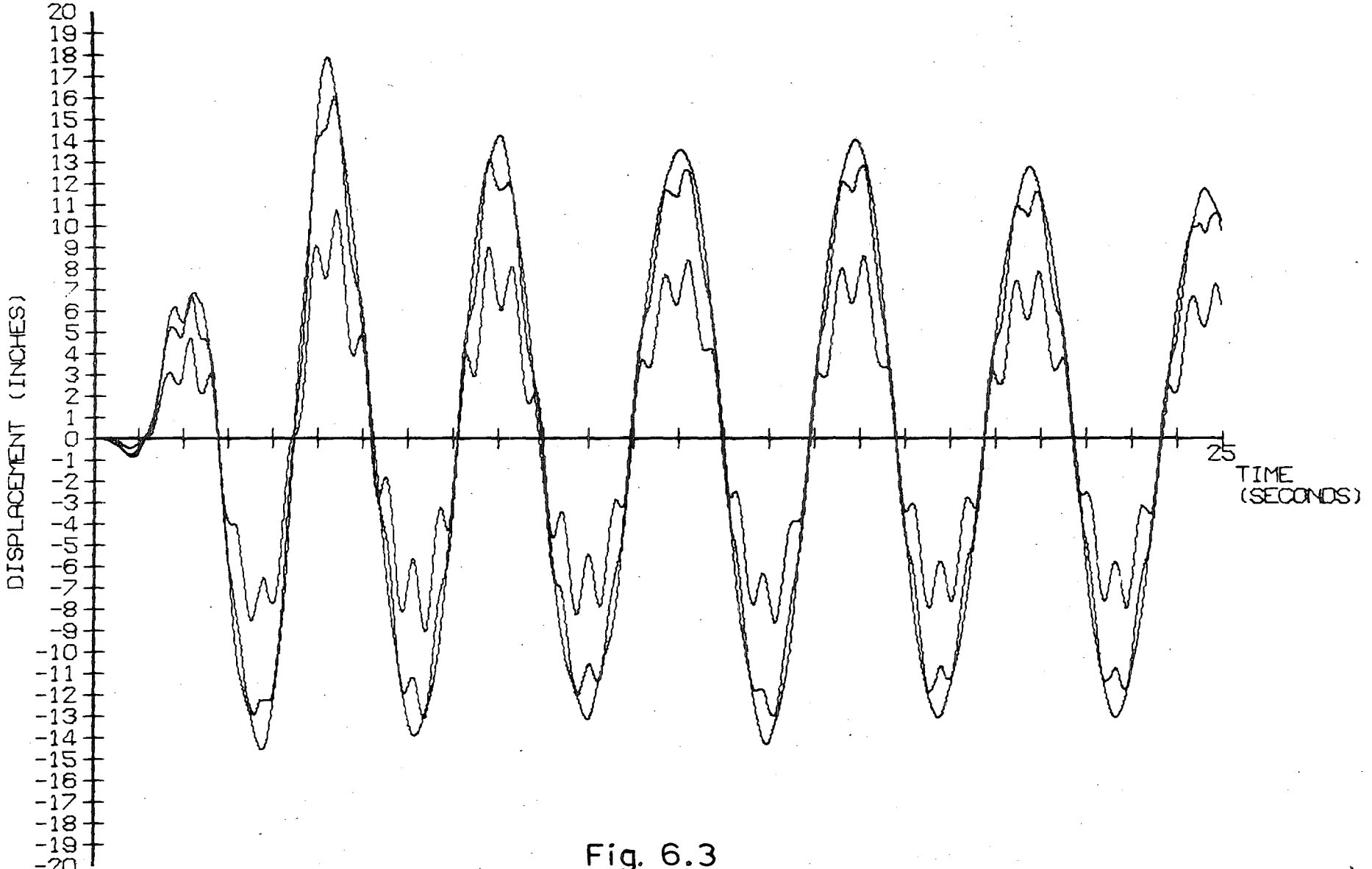
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Fig. 6.3

A second analysis, similar to the first one except that the 2% damping coefficients were read in to the XBFRME program, resulted in the displacement/time history shown in Figure 6.4 being produced. Maximum displacements in this case are

Mass 1 : 9.364 ins at 5.441 secs
Mass 2 : 14.35 ins at 5.391 secs
Mass 3 : 16.53 ins at 5.234 secs

Once again this result is identical to that given by an ELRES analysis.

When the Moran and Cheney bracing with no prestress was incorporated and allowed to act in an extended elastic manner the undamped response of the frame, supporting the full tank with no convective action of the water, to the El Centro N/S accelerogram factored by 1.0, as presented in Figure 6.5 was determined.

In this case the maximum displacements are

Mass 1 : 3.934 ins at 23.58 secs
Mass 2 : 6.717 ins at 23.57 secs
Mass 3 : 8.901 ins at 24.26 secs

Inclusion of 2% damping in this analysis was next undertaken. In the first place the damping matrix was calculated using the effective total lateral stiffness characteristics of the braced frame and was then overwritten into the program XBFRME. The resulting response is shown in Figure 6.6.

Maximum displacements in this case are:-

Mass 1 : 1.978 ins at 14.58 secs
Mass 2 : 3.391 ins at 14.58 secs
Mass 3 : 4.504 ins at 14.58 secs

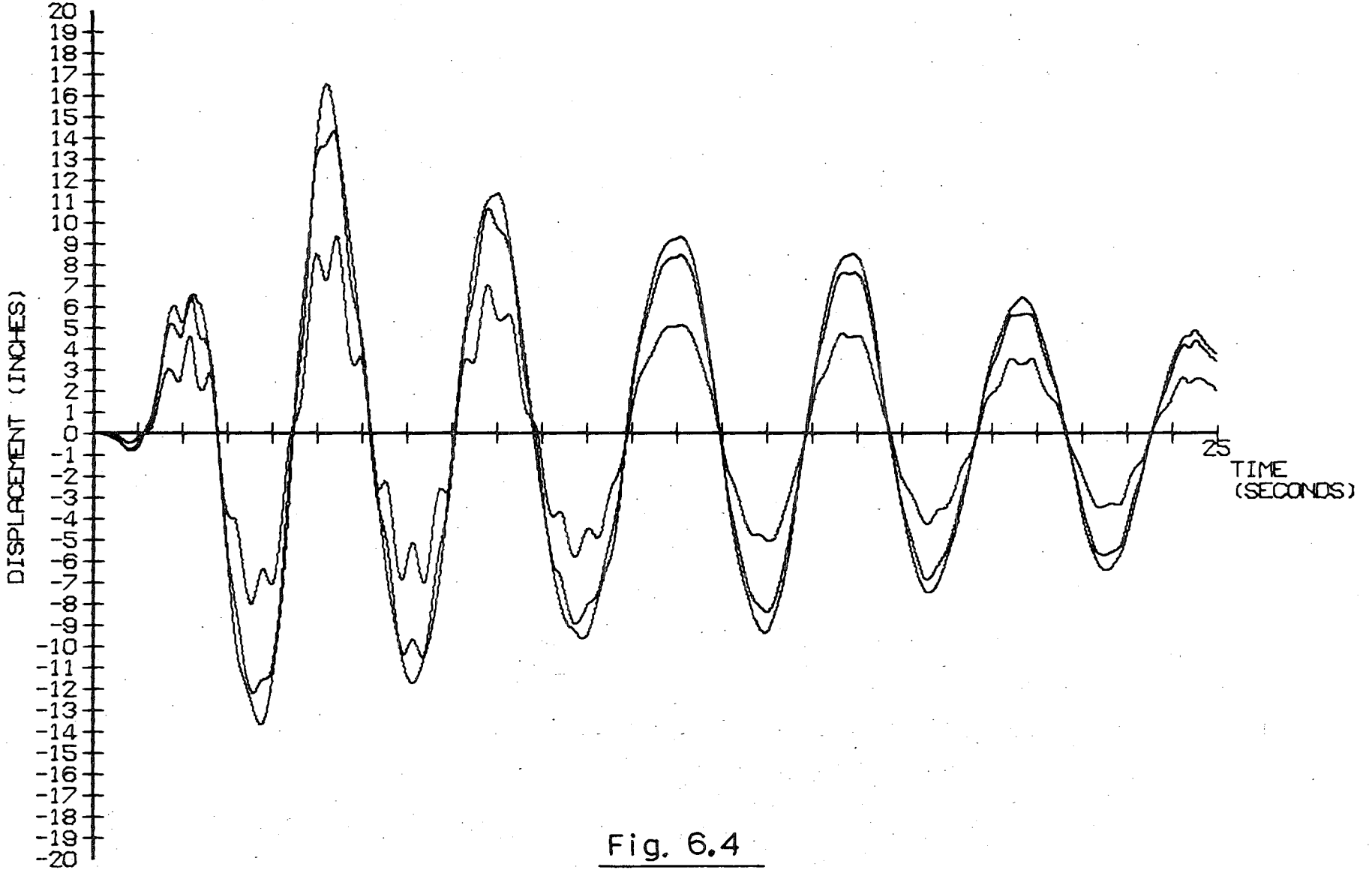
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Fig. 6.4

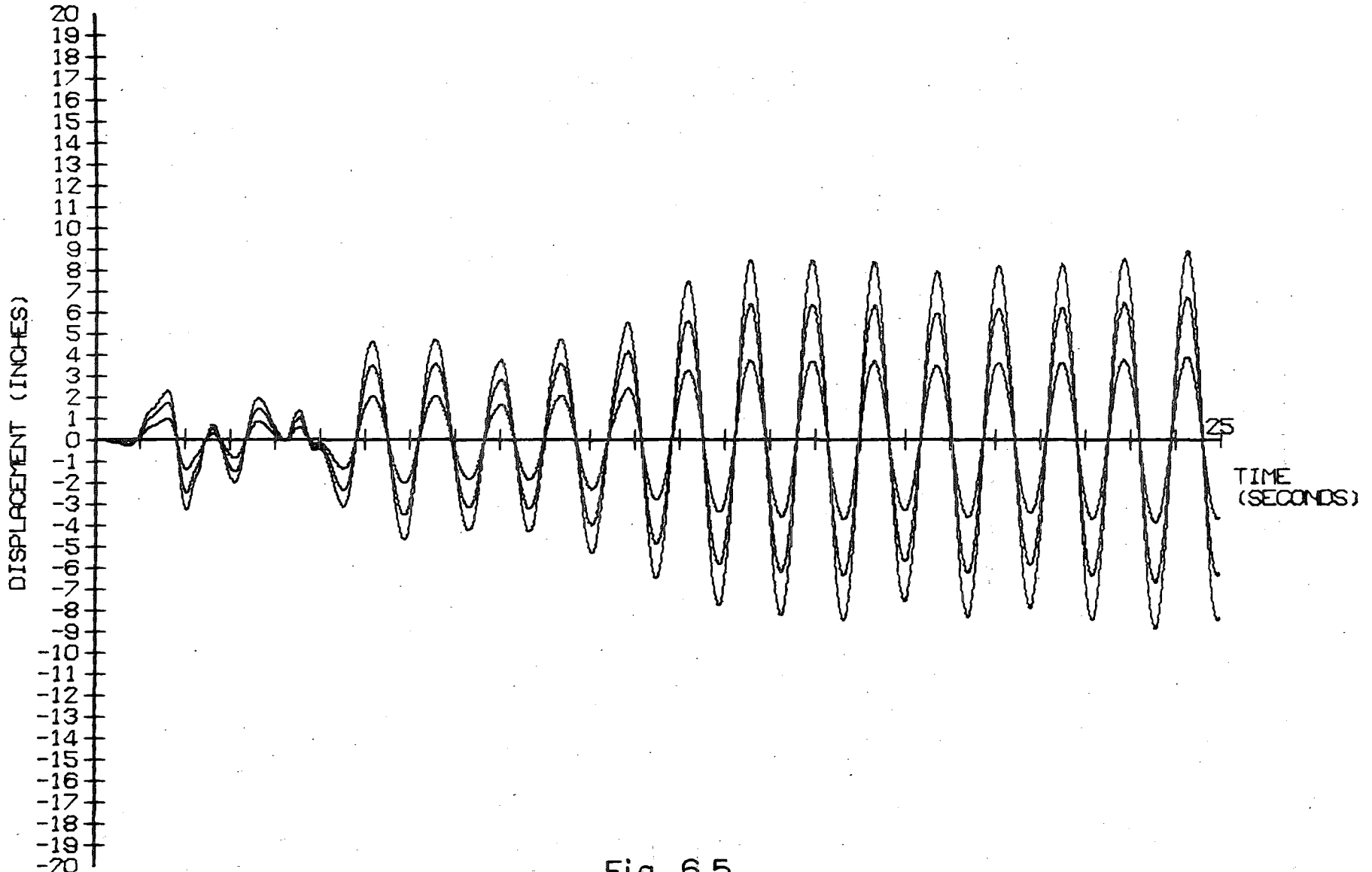
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Fig. 6.5

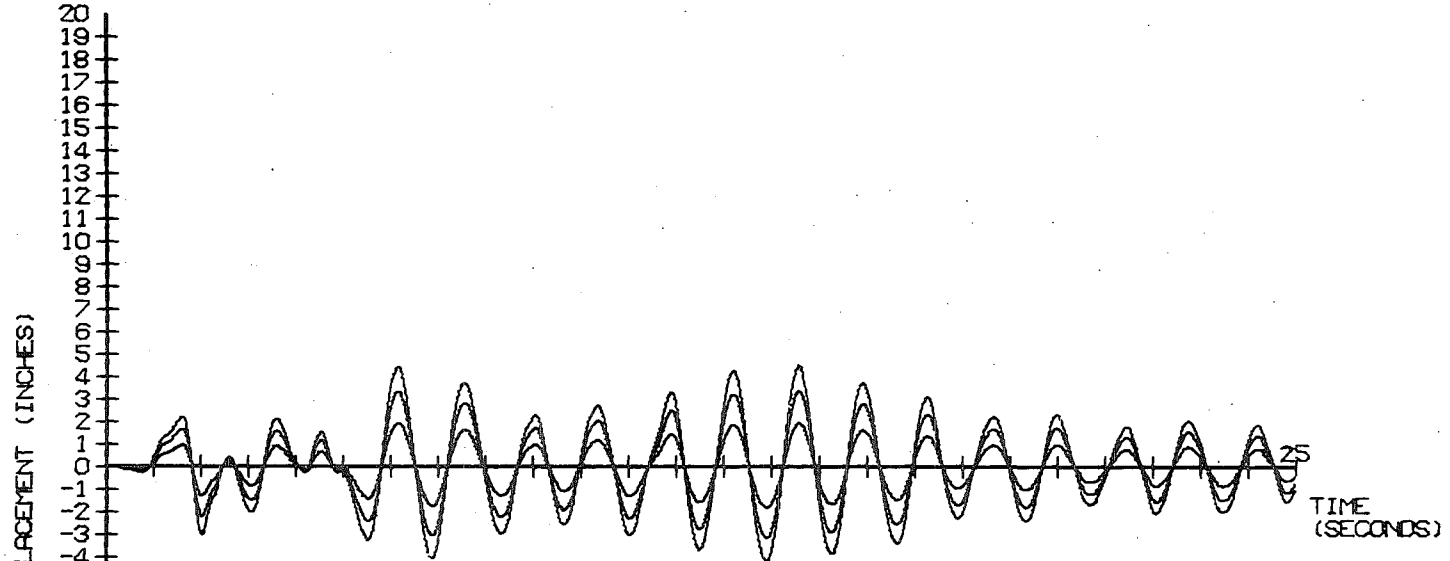
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Fig. 6.6

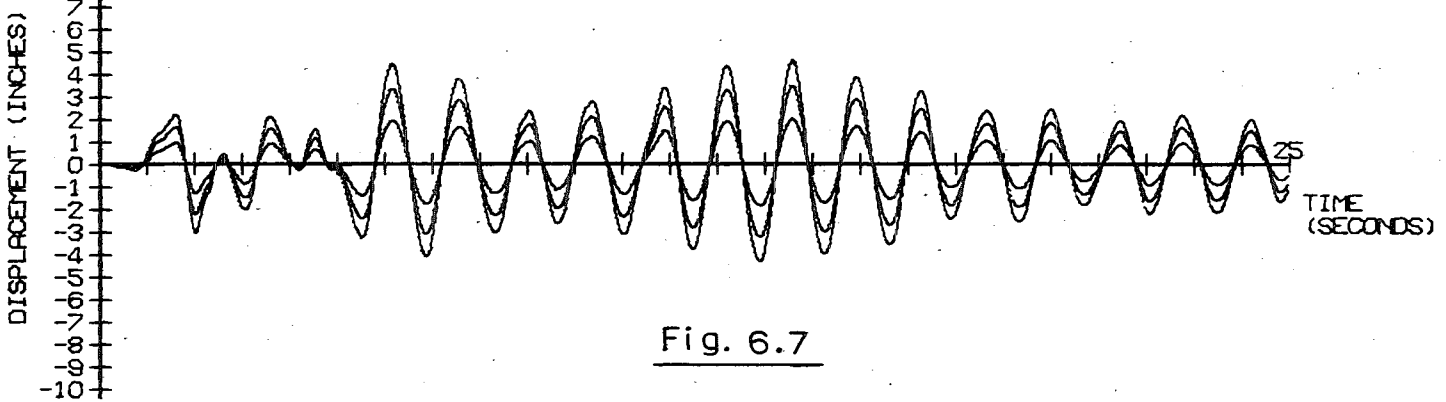
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Fig. 6.7

A further analysis, similar to the previous one except that the 2% damping matrix was determined internally in the XBFRME program (and therefore based on the unbraced frame lateral stiffness matrix) after the damping coefficients had been read in, resulted in the displacement/time plot shown in Figure 6.7.

Maximum displacements in this case are:-

Mass 1 : 2.047 ins at 14.57 secs
Mass 2 : 3.500 ins at 14.57 secs
Mass 3 : 4.644 ins at 14.58 secs

The responses of Figures 6.5 and 6.6 were again confirmed by an independent ELRES analysis of the system.

The effect of sealing the El Centro accelerogram by 1.5 instead of 1.0 was examined by applying this excitation to the 2% damped extended elastic braced frame supporting a full tank (with no allowance for the convective action of the water) and the response shown in Figure 6.8 was obtained.

The maximum displacements are 50% greater than in the previous analysis viz:

Mass 1 : 3.07 ins at 14.57 secs
Mass 2 : 5.25 ins at 14.57 secs
Mass 3 : 6.96 ins at 14.58 secs

The 2% damped extended elastic response, to the El Centro N/S accelerogram factored by 1.0, of the Moran and Cheney braced frame supporting half a tank full of water acting as a solid mass, is shown in Figure 6.9. The maximum displacements are:-

Mass 1 : 2.744 ins at 4.418 secs
Mass 2 : 4.686 ins at 4.418 secs
Mass 3 : 6.198 ins at 4.418 secs

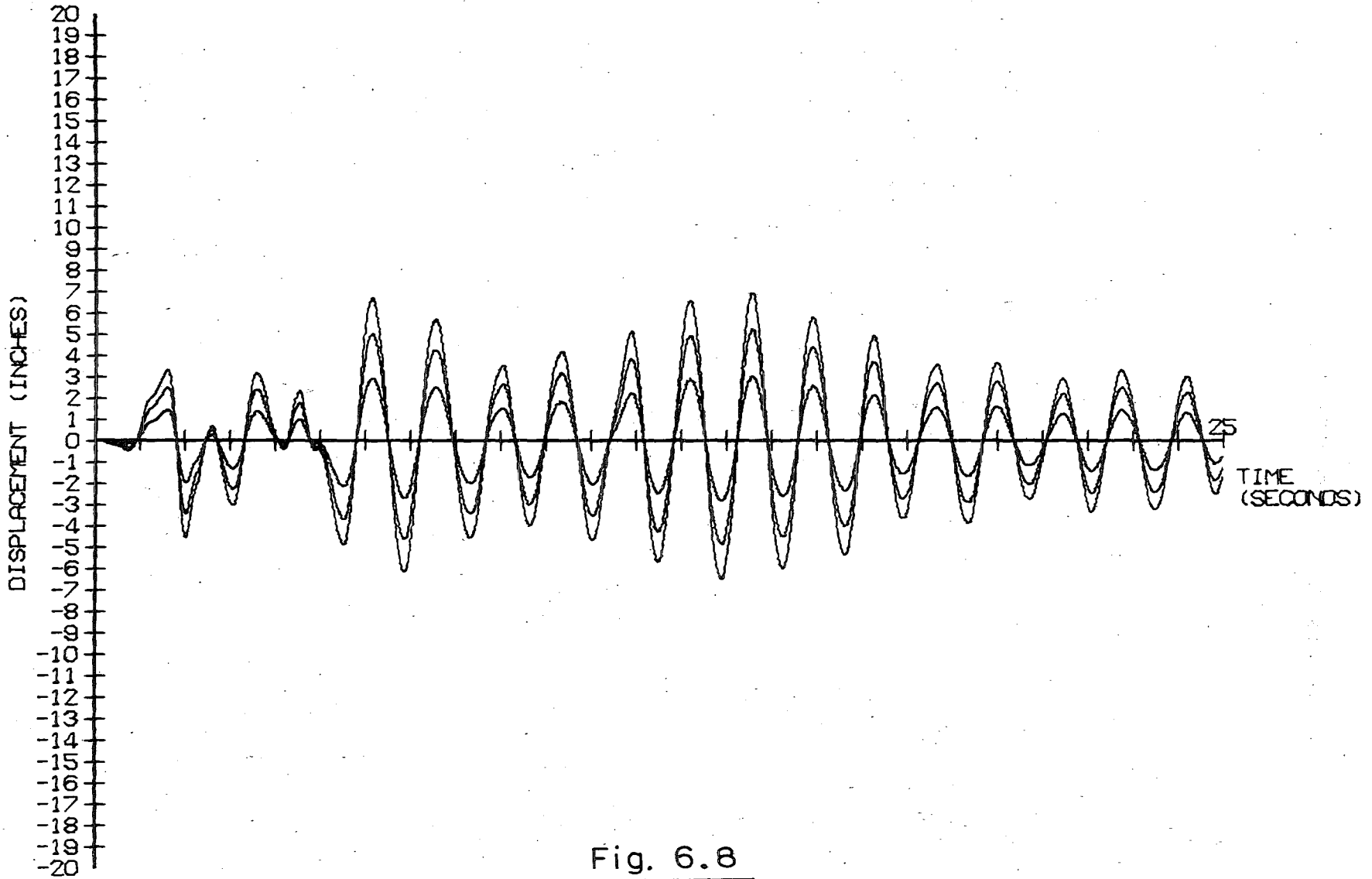
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Fig. 6.8

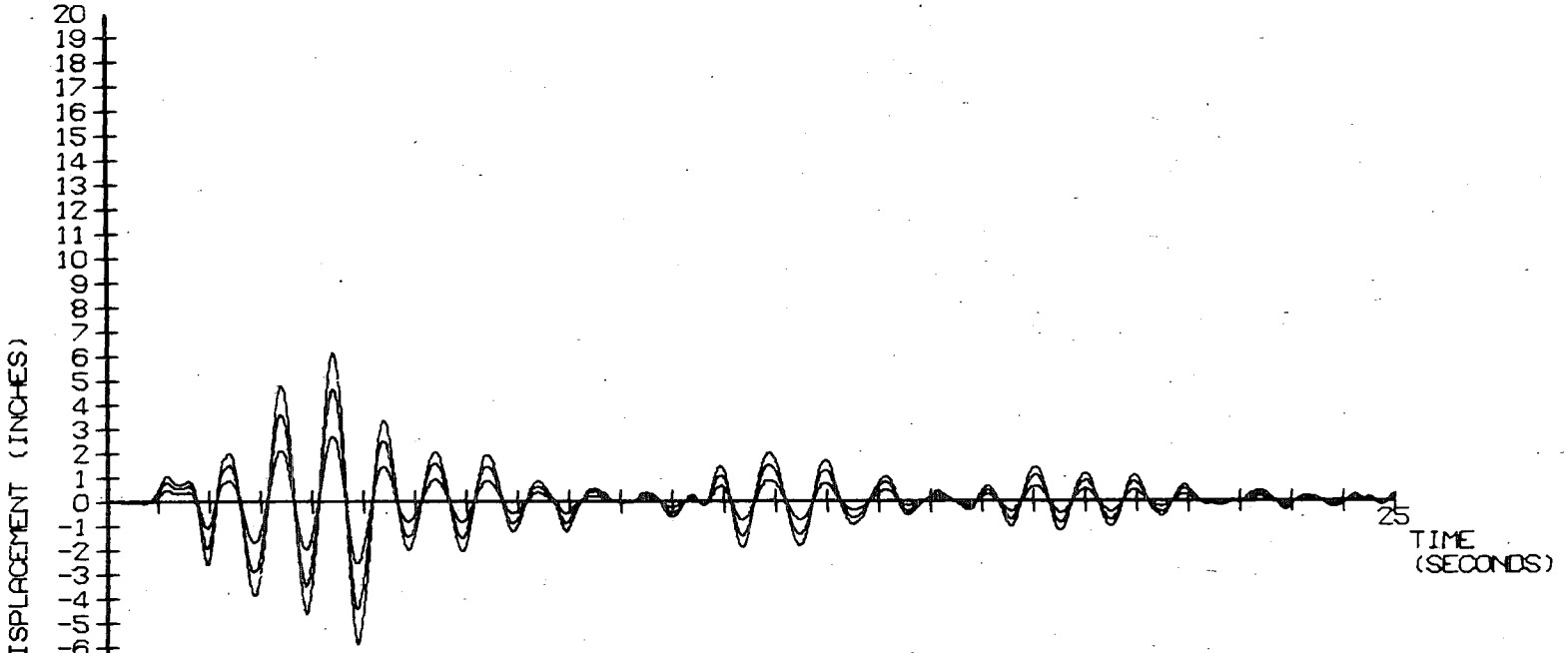

Fig. 6.9

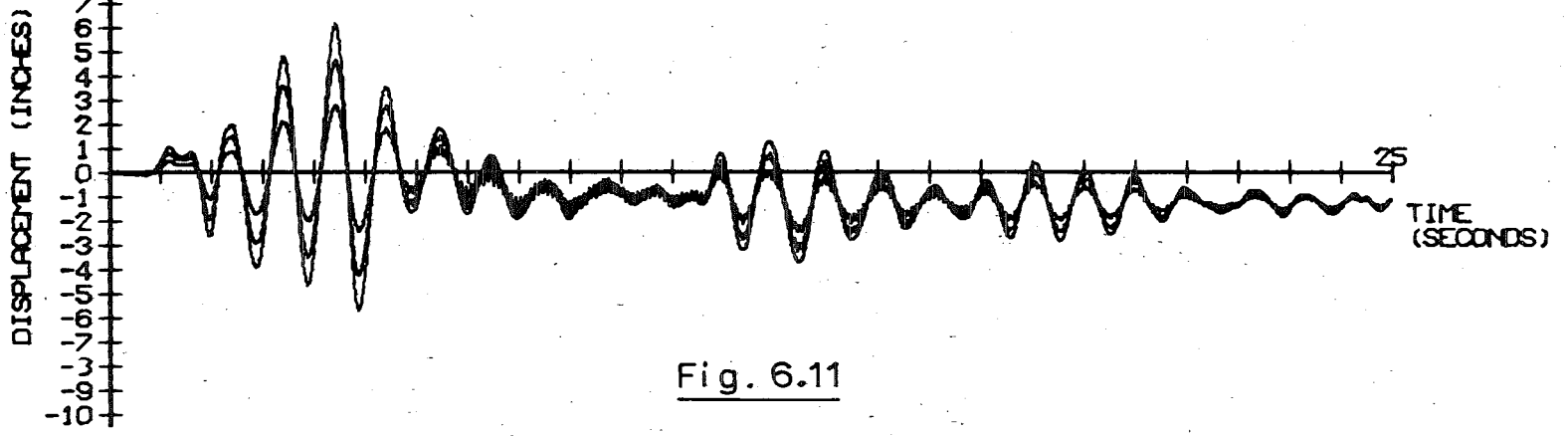

Fig. 6.11

The 2% damped response to the same excitation when the Moran and Cheney frame, having the mild steel bracing rods pretensioned to one quarter of their yield value and the previously established post yield axial stiffness incorporated, supports a full tank acting as a rigid mass is presented in Figure 6.10. The maximum displacements are:-

Mass 1 : 8.004 ins at 6.215 secs
Mass 2 : 9.252 ins at 6.199 secs
Mass 3 : 9.835 ins at 6.180 secs

and the total yield displacements suffered by the bottom bay braces are 6.058 ins and 4.931 ins respectively.

In Figure 6.11 the response of half a tank of water acting as a rigid mass in an otherwise similar system subject to the same excitation, is shown. The maximum displacements are:-

Mass 1 : 2.818 ins at 4.437 secs
Mass 2 : 4.704 ins at 4.430 secs
Mass 3 : 6.198 ins at 4.418 secs

Total yield displacements of 0.223 ins and 0.027 ins occur in the bottom bay braces.

VI.4 Digital Computer Analyses Including Convective Action of the Water

When the fluid in the elevated tank is represented by a two mass system, the incorporation of Step C (section V.7) is necessary in the analysis procedure. In the case of the tank used in this study the results of a MODANAL analysis, for various water depths, is presented in Table 4.

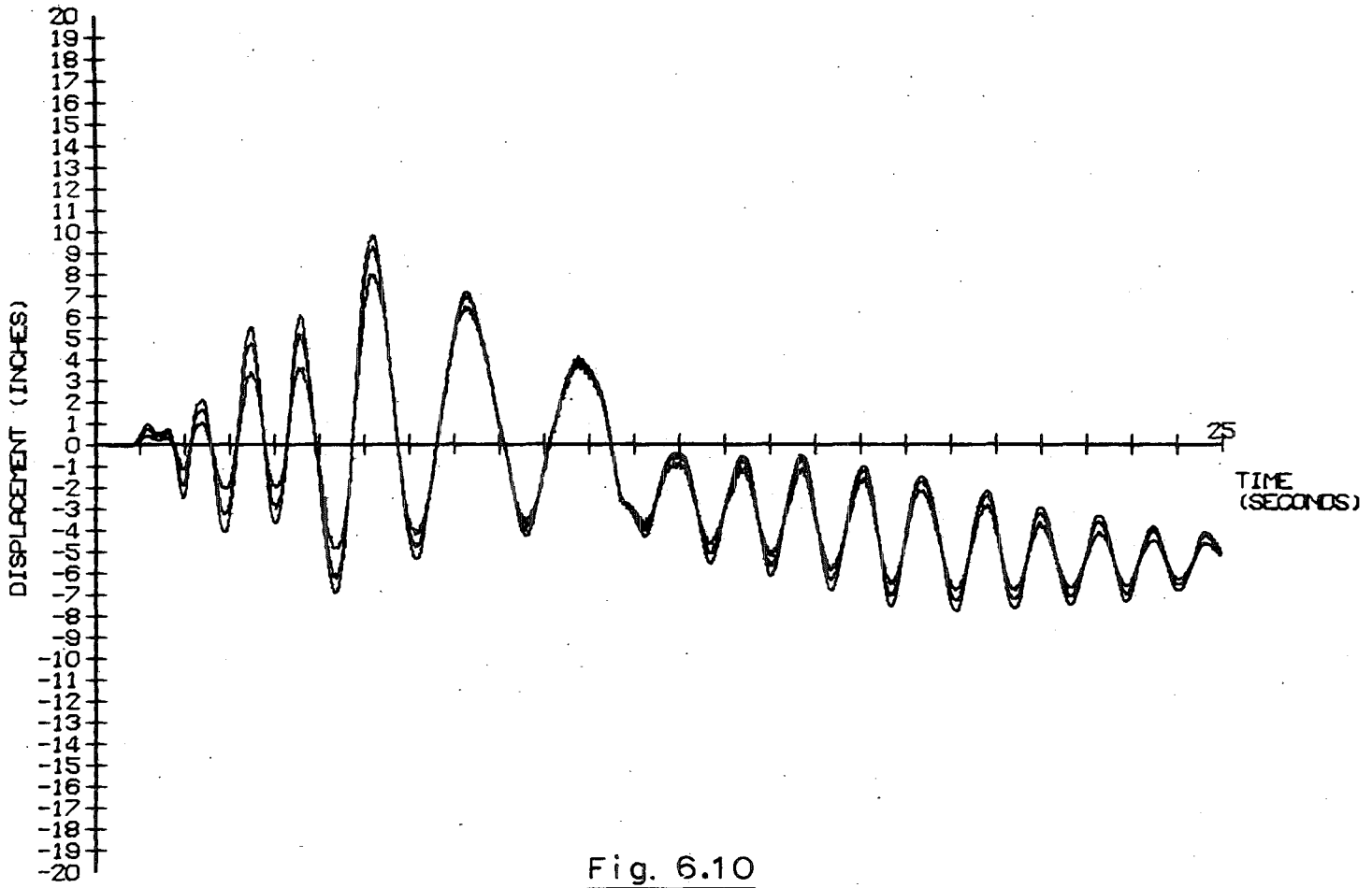
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Fig. 6.10

Water Depth (ft)	m_{so} (lb.wt)	m_1 (lb.wt)	k_1 (lb.wt/ft)
18.0	640,000	246,000	36,310
13.5	256,000	238,000	33,530
9.0	250,000	210,000	25,810
4.5	145,000	134,000	10,600

Table 4

Using these results the equivalent four mass system (Figure 4.2) for each depth of water may be set up. The use of the '2D-Structure' program (Appendix IV) then enables a flexibility matrix for each system to be determined. For the full tank case this has the form

$$10^{-4} \times \begin{bmatrix} 5.773 & 7.731 & 7.480 & 7.971 \\ 7.731 & 12.219 & 12.472 & 13.301 \\ 7.480 & 12.472 & 13.957 & 14.913 \\ 7.971 & 13.301 & 14.913 & 58.433 \end{bmatrix}$$

Use of the matrix inversion program (LIST LP3, Appendix IV) enables the corresponding lateral stiffness matrix to be calculated, viz:-

$$10^{4} \times \begin{bmatrix} 1.34 & -1.31 & 0.451 & -0.0001 \\ -1.31 & 2.21 & 1.27 & 0.0006 \\ 0.444 & -1.27 & 0.99 & -0.026 \\ 0.0056 & -0.0049 & -0.023 & 0.024 \end{bmatrix}$$

and subsequently the normal mode analysis (LIST LP2) and damping matrix formation (LIST LP 2A) programs may be used to determine the damping coefficients corresponding to 2% critical damping required as input for the XBFRME program

viz: CG = 0.03851

and CR = 0.01023 for the full tank case.

Similar necessary input data can be prepared for other depths of water in the tank to enable the XBFRME program to be used to determine the seismic response of alternative four mass systems.

The 2% damped response of Moran and Cheney's tower, acting in the extended elastic range and supporting a full tank of water free to act convectively, to the El Centro digitised accelerogram factored by 1.0 is shown in Figure 6.12. The maximum displacement values are:-

	Figure 6.12	whereas in Figure 6.7
Mass 1 :	3.776 ins at 17.91 secs	2.047 ins at 14.57 secs
Mass 2 :	6.458 ins at 17.91 secs	3.500 ins at 14.57 secs
Mass 3 :	8.561 ins at 17.91 secs	4.644 ins at 14.58 secs

the displacements of Figure 6.7 being those of an identical structural system but with no allowance for the convective action of the water (i.e. a three mass model instead of the four mass one). The half full tank response, in an otherwise identical analysis is shown in Figure 6.13. The maximum displacements are:-

Mass 1 : 2.441 ins at 13.65 secs
Mass 2 : 4.161 ins at 13.65 secs
Mass 3 : 5.477 ins at 13.66 secs

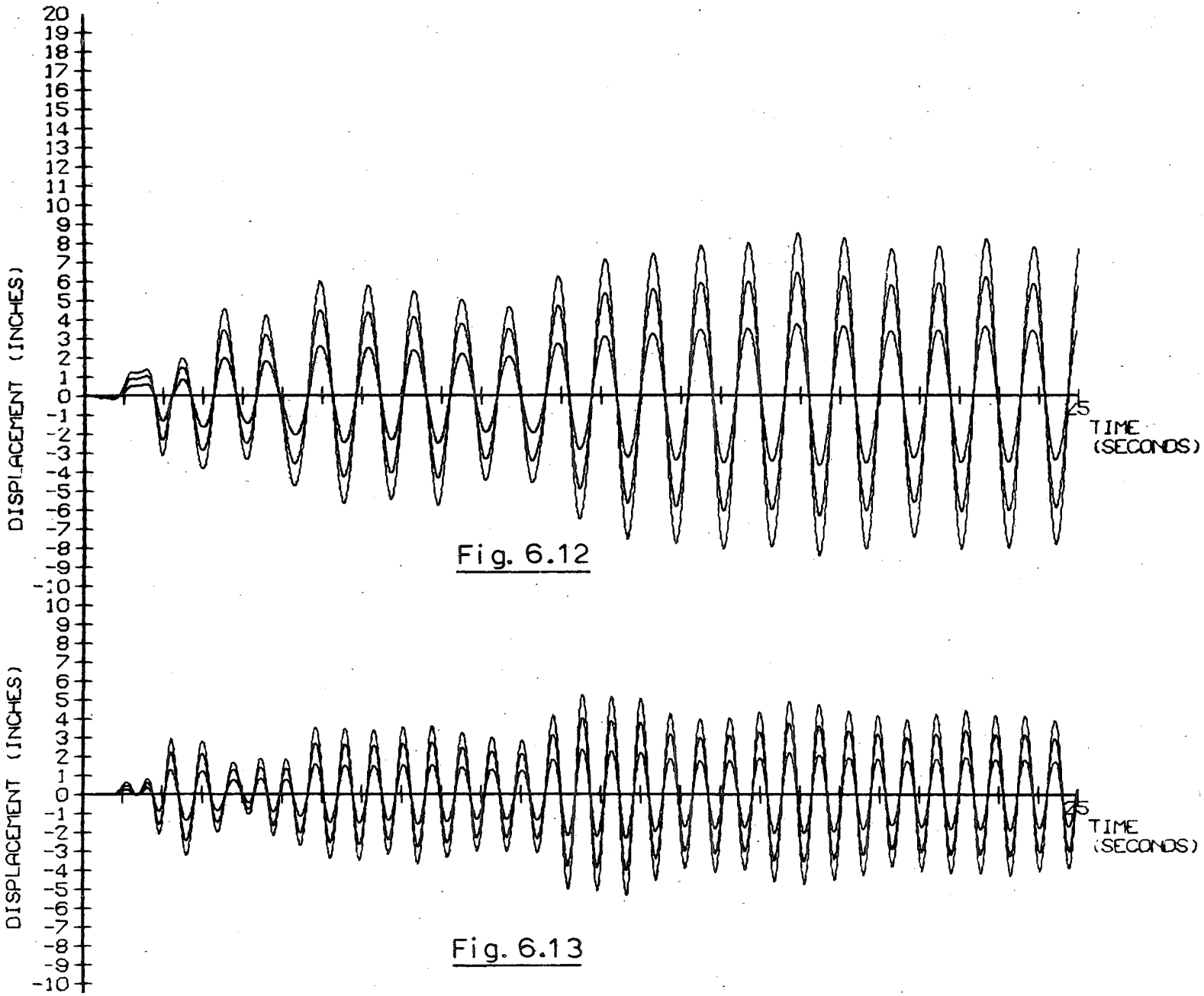

Fig. 6.12

Fig. 6.13

When the full water tank is represented by a two mass system, and the Moran and Cheney braced tower has the mild steel bracing rods prestressed to one quarter of their yield level and their post yield axial stiffnesses set at the values determined earlier using the TRUEFORCE program, the 2% damped responses to the El Centro accelerogram, factored by 1.0 and 0.6 respectively, are shown in Figures 6.14 and 6.15. The maximum displacements are:-

	Figure 6.14		Figure 6.15
Mass 1 :	5.042 ins at 4.422 secs	and	3.508 ins at 7.254 secs
Mass 2 :	6.549 ins at 4.414 secs		4.510 ins at 7.250 secs
Mass 3 :	7.729 ins at 4.398 secs		5.169 ins at 7.230 secs

The total yield displacements suffered by the bottom bay braces are 3.096 ins and 2.697 ins in the first case and 1.562 ins and 1.200 ins in the second one.

In Figure 6.16 the 2% damped convective response of a full water tank system to a similar excitation, when the Moran and Cheney braced tower has no prestress in the diagonal members, is presented. The maximum displacements are:-

Mass 1 : 6.600 ins at 14.52 secs
Mass 2 : 8.243 ins at 14.52 secs
Mass 3 : 9.425 ins at 14.53 secs

and total yield displacement of 4.005 ins and 1.433 ins are experienced by the bottom bay braces.

In Figures 6.17 and 6.18 the responses are shown of systems in which the structural bracing has been selected to exhibit multiphase characteristics, including fracturing of high tensile steel members. In both cases the bracing was similar to that of the Moran and Cheney frame, but prestress forces equal to one quarter of the yield loads

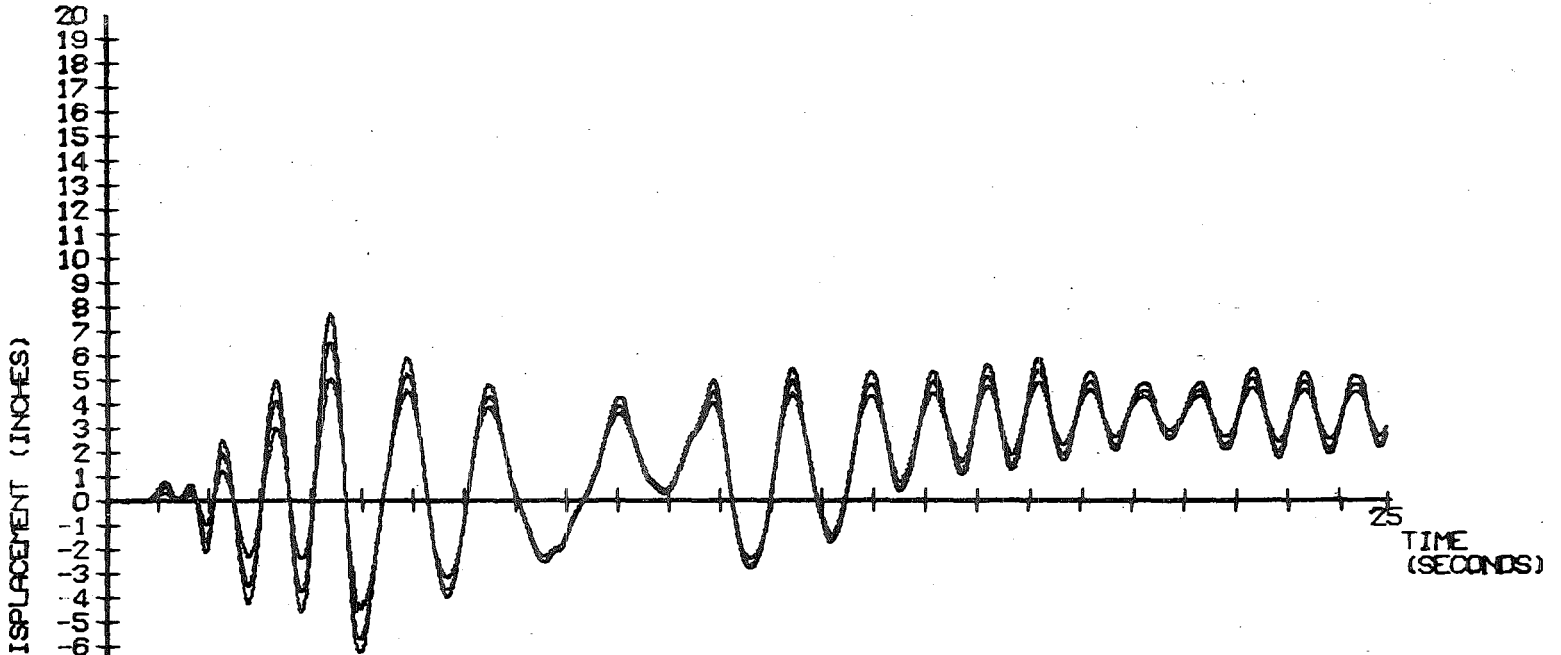

Fig. 6.14

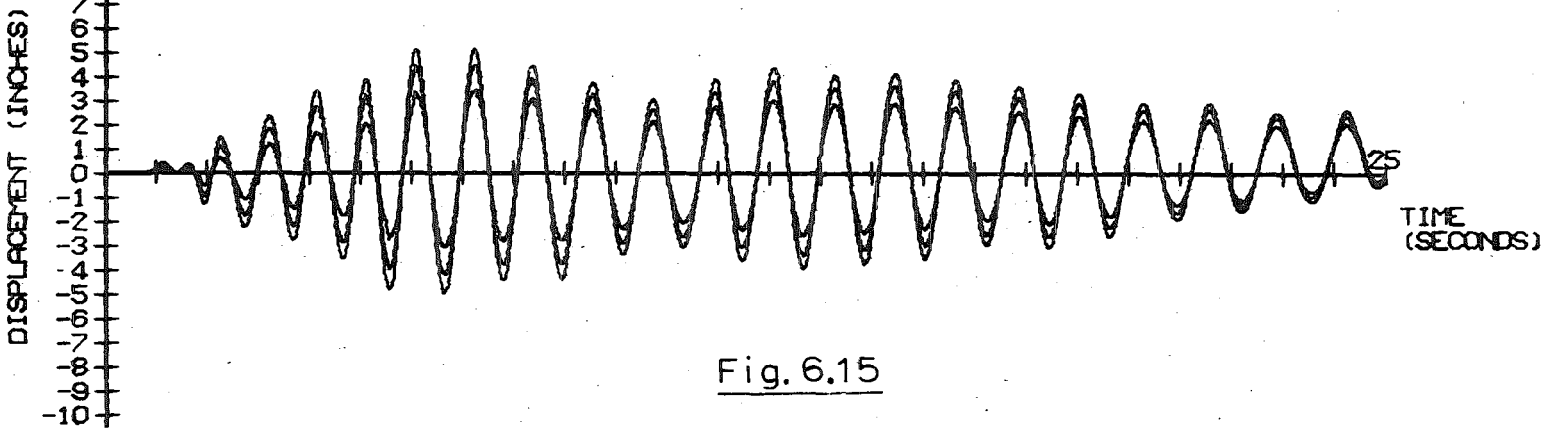

Fig. 6.15

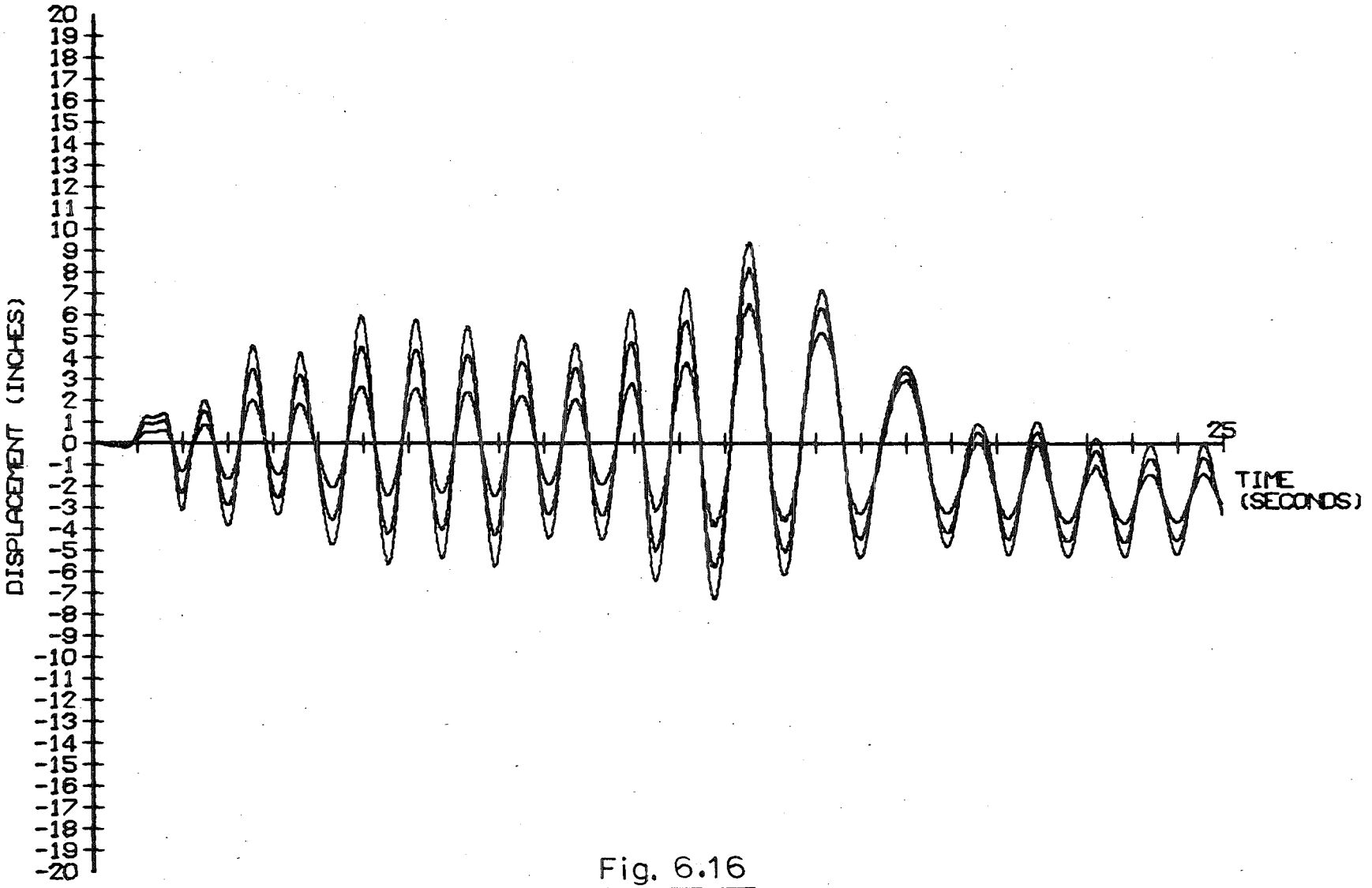
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Fig. 6.16

were incorporated. However, the bottom bay bracing was considered to be composed of the equivalent of 2 in^2 high tensile steel and 2.82 in^2 mild steel diagonals instead of the original total of 4.82 in^2 of mild steel bracing. As the high tensile steel fractured at about three seconds after the commencement of the accelerogram the effect was to produce a flexible lower bay for the remainder of the response. Figures 6.17 presents the displacement/time history for the full tank case and that of the half full tank case is shown in Figure 6.18. In both cases convective action of the water is taken into account. The maximum displacements are:-

	Figure 6.17		Figure 6.18
Mass 1 :	7.438 ins at 22.89 secs	and	7.791 ins at 15.14 secs
Mass 2 :	7.753 ins at 22.88 secs		8.115 ins at 15.14 secs
Mass 3 :	7.954 ins at 20.29 secs		8.311 ins at 15.16 secs

Total yield displacements in the bottom bay braces were calculated to be 4.475 ins and 5.500 ins in the first case and 5.853 ins and 2.565 ins in the second one.

When half the Moran and Cheney (prestressed) mild steel bracing was combined, in parallel, with selected high tensile steel bracing in the basic frame the 2% damped responses of the system, allowing for a full tank of water which acts convectively, to the El Centro accelerogram are presented in Figures 6.19, 6.20, 6.21 and 6.22.

The system responding as shown in Figure 6.19 has the equivalent of two 0.25 in. diameter high tensile steel rods in each bay, pre-stressed to one quarter of their ultimate strength, whereas the system corresponding to the response shown in Figure 6.20 has 0.5 in. diameter high tensile steel rods in place of the 0.25 in. diameter

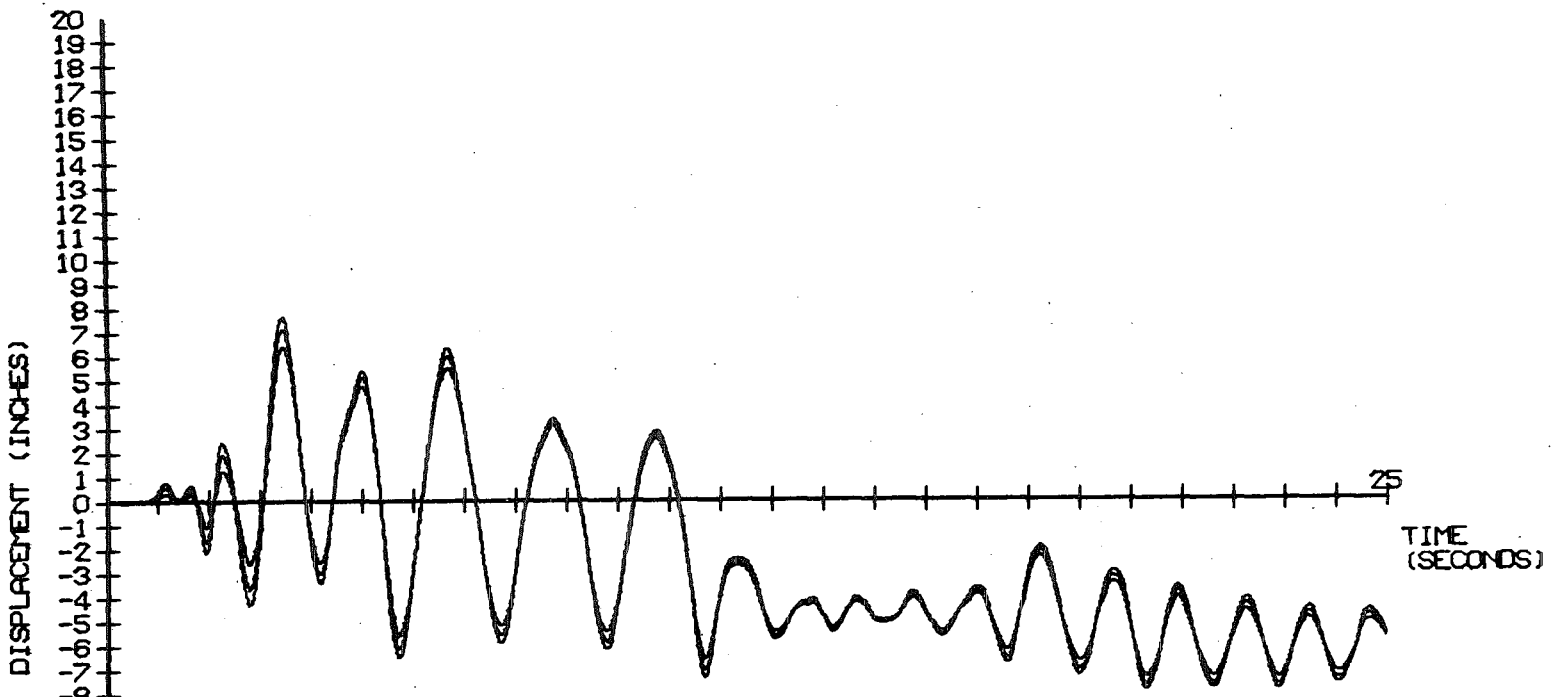

Fig.6.17

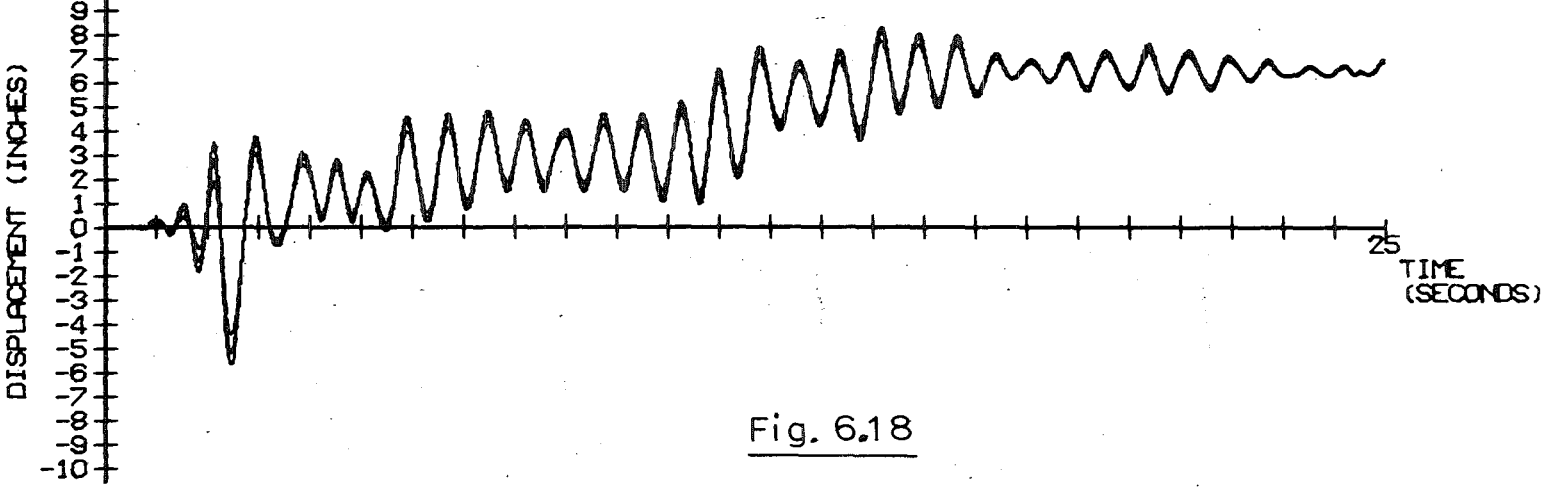

Fig. 6.18

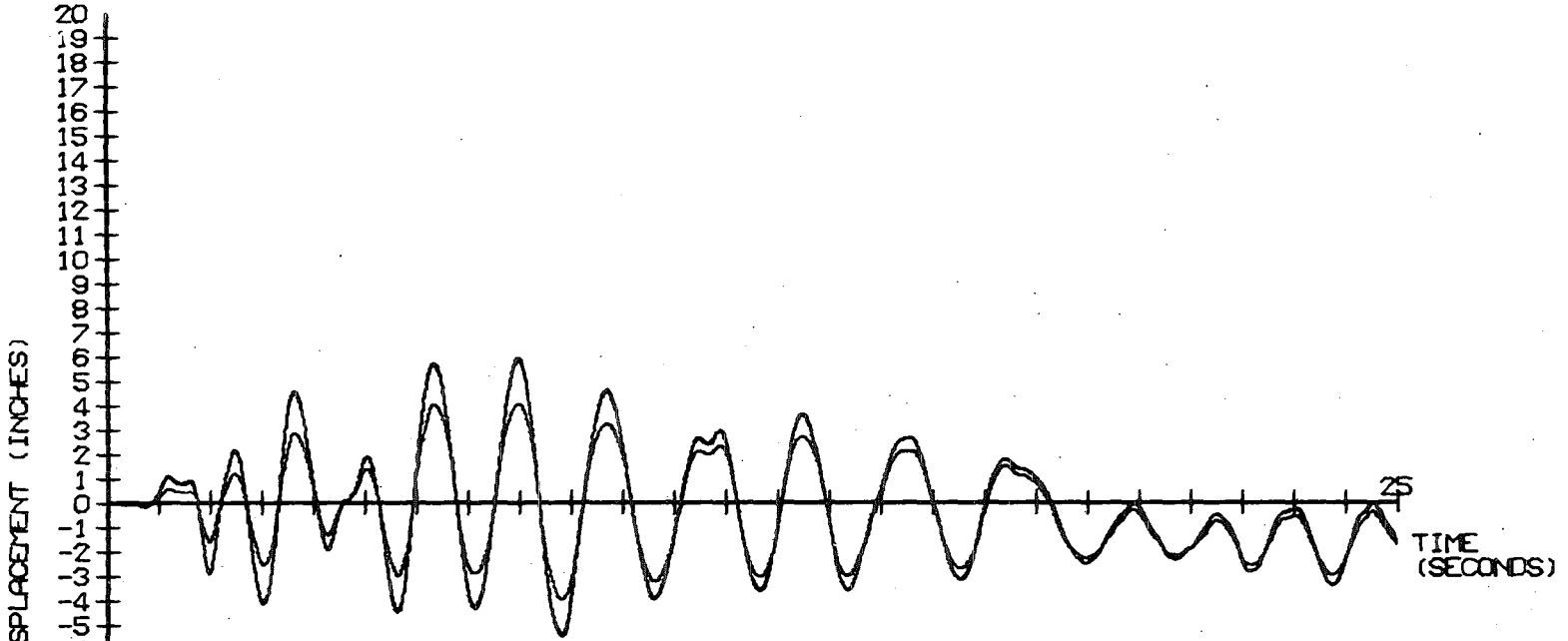

Fig. 6.19

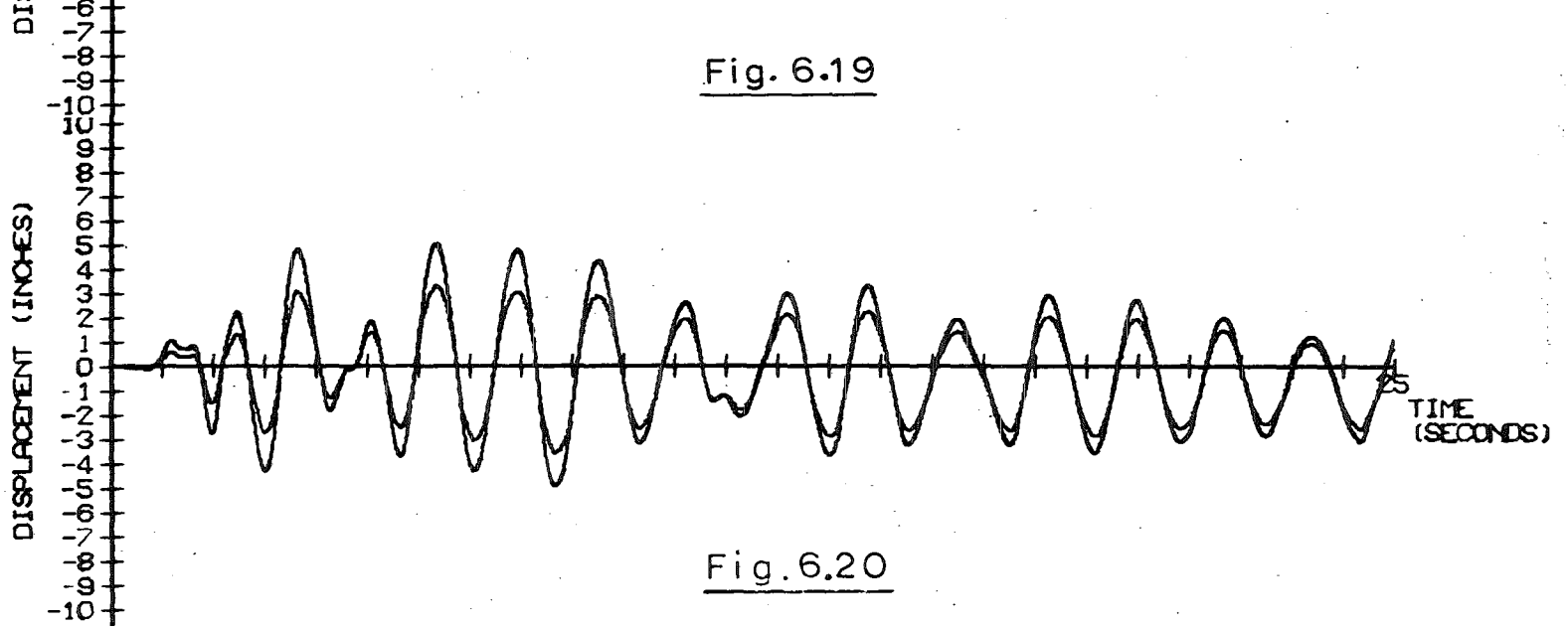

Fig. 6.20

ones. The maximum displacements are:-

	Figure 6.19		Figure 6.20
Mass 1 :	4.070 ins at 7.977 secs	and	3.650 ins at 8.664 secs
Mass 2 :	5.875 ins at 7.988 secs		5.022 ins at 6.344 secs
Mass 3 :	5.964 ins at 7.988 secs		5.102 ins at 6.340 secs

with bottom bay bracing yield extensions of 2.127 in. and 2.072 in. respectively in the first case and 1.413 in. and 1.706 in. in the second one.

The responses of two similar systems but with 0.75 in. diameter and 1.0 in. diameter high tensile steel rods respectively replacing the 0.5 in. diameter ones of the last system are presented in Figures 6.21 and 6.22. The maximum displacements are:-

	Figure 6.21		Figure 6.22
Mass 1 :	3.759 ins at 3.684 secs	and	4.969 ins at 24.61 secs
Mass 2 :	5.193 ins at 3.672 secs		5.645 ins at 13.39 secs
Mass 3 :	5.265 ins at 3.672 secs		5.714 ins at 13.39 secs

with bottom bay bracing yield extensions of 1.816 in. and 1.214 in. in the first case and 3.026 in. and 1.899 in. in the second one.

In all of the last four systems at least two of the high tensile steel rods fractured during the response.

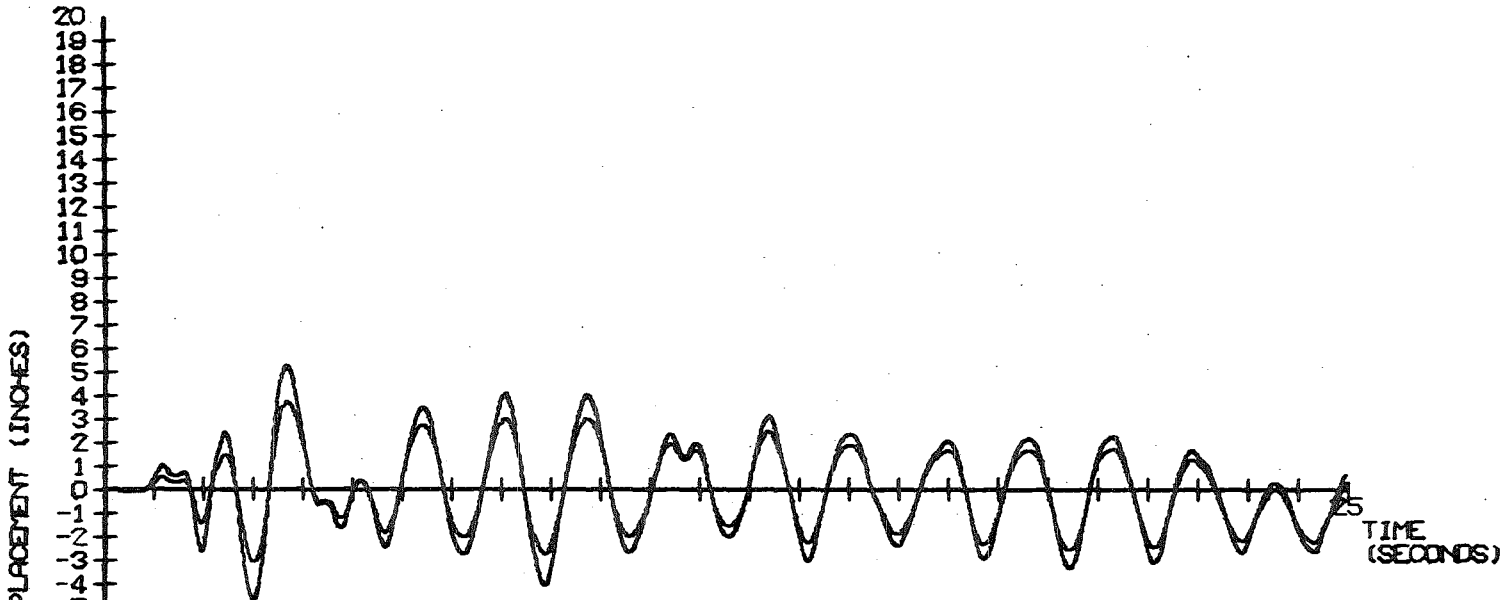

Fig. 6.21

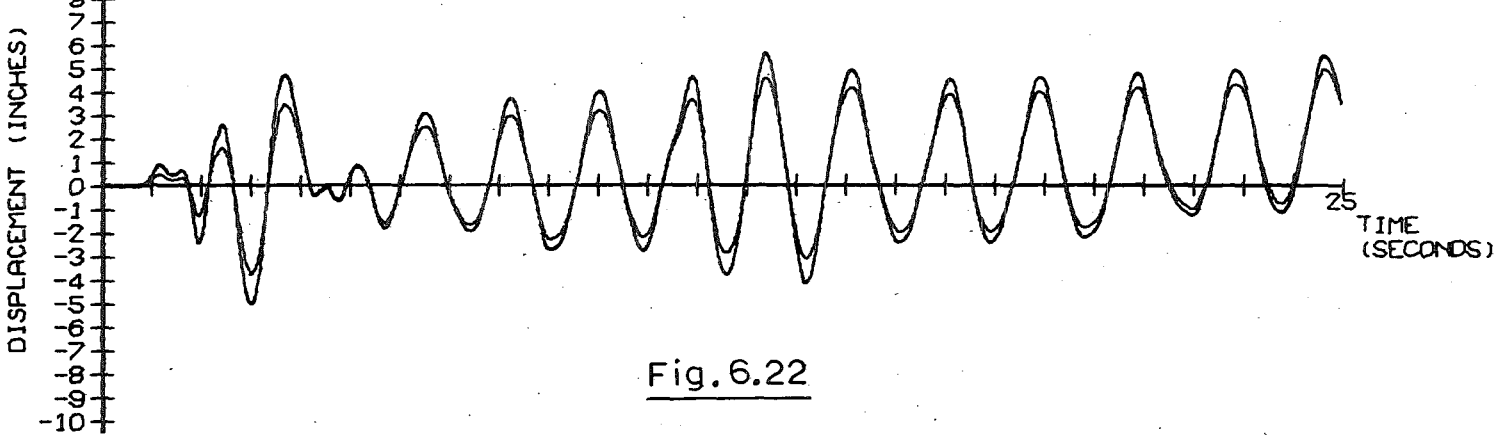

Fig. 6.22

CHAPTER SEVEN

REVIEW OF ANALYSES UNDERTAKEN AND COMMENTS ON APPLICABILITY OF THE PROPOSED ANALYSIS METHOD

VII.1 Review of the Analyses Described in Chapter Six

The undamped extended elastic response shown in Figure 6.3 of an empty tank on the unbraced Moran and Cheney frame, to the El Centro N/S accelerogram, is itself of limited interest apart from serving as a confirmation of the correct functioning of the integration process in the XBFRME program by virtue of the response being identical to that derived using the ELRES program. Some evidence of beating of the first and second masses is evident in the plot. However when Figure 6.4 is compared with Figure 6.3 the effect of the inclusion of 2% of critical viscous damping in the response calculation is apparent. Since it would be unrealistic to take no account of damping in seismic response calculations, some small allowance is normally included and on the basis of previous tests[21] and of the result presented in Chapter Two, 2% appears to be a reasonable figure to use for the elevated water tower type of structure. Independent determination of the displacement/time response using the ELRES program confirmed that the damping section of XBFRME functions correctly.

The extended elastic responses to the same excitation of a full tank (with no allowance for the convective action of the

contents) on a Moran and Cheney frame braced as in the original structure[21] are presented in Figure 6.5 for the undamped situation and in Figure 6.6 for the 2% damping case. The necessity to include some damping if the prototype system is to be accurately modelled is even more evident as a result of this comparison. The continued build-up of displacements with time beyond 20 seconds in the undamped case does not correlate with the recorded motion of structures in earthquakes whereas the plot in Figure 6.6 does represent the observed movements more reasonably. However a problem does arise in the formation of the damping matrix referred to in Chapter Four (equation 4(7)) when the stiffness of the system is not constant throughout the response calculation. The results presented in Figure 6.6 were calculated using a damping matrix formed with equation 4(7), the total initial frame stiffness (i.e. including the bracing contribution) being used as the term [K]. The damping matrix was overwritten into the program XBFRME for this particular analysis whereas in its normal operation the program XBFRME calculates the damping matrix on the basis of the unbraced frame stiffness. When the normal procedure was followed, that is the 2% damping matrix was assembled by the XBFRME provision based on FK rather than TK in the notation of the program - the response plot shown in Figure 6.7 was obtained. The top mass displacement is 3% greater in the second case and this reflects the effectively lower damping allowance made. The true response of a system having its stiffness varying between the TK and FK values will lie somewhere between the plots of Figure 6.6 and 6.7. Provision to calculate the damping matrix at each change of stiffness could have been

incorporated in the XBFRME program but in view of the small differences in response which were determined in the checks on the two extreme cases this provision was considered to be unnecessary. In fact all subsequent analyses were carried out using the XBFRME damping matrix determination based on the unbraced frame stiffness.

In Figure 6.8 the displacement/time plot is shown of an identical system to that having the response presented in Figure 6.7, but on this occasion the exciting accelerogram was factored by 1.5. The plot confirms that the factoring of the amplitude of the excitation merely affects the displacement of the response of the elastic system in a linear manner.

The response of an identical structural system supporting half a tank full of water (with no allowance for convective action) instead of a full tank is shown in Figure 6.9. The accelerogram was factored by 1.0 to produce this plot and so the response is directly comparable with that of Figure 6.7, the full tank case. The larger, and earlier, response of the lighter system to the particular excitation used is evident from a comparison between these figures.

The first multiphase response plot is that presented in Figure 6.10, being the 2% damped response to the El Centro N/S accelerogram of a full tank (with no allowance for convective water action) supported on a Moran and Cheney frame having the original mild steel bracing prestressed to one quarter of the yield level and having the post yield axial stiffness of the braces set at the values determined using the TRUEFORCE program (Chapter Three), assuming that the material

exhibits similar properties to the mild steel of the rods used in the tests described in Chapter Three. The plot is limited to 25 seconds from the commencement of the excitation, but it is evident that this does include the maximum amplitude. From the typed output of the XBFRME program it may be determined that substantial yielding in the bottom bay braces occurs in the 3 to 6 seconds section of the response and that the motion in the subsequent 5 seconds is essentially that of a frame unbraced in the bottom bay - with consequent lengthened period - whereas from about twelve seconds onwards the response corresponds to that of a frame cross-braced elastically in the upper bays but with one elastic brace active for part of the time in the bottom bay. The bracing extensions in the post elastic range correspond to ductility requirements of 5.7 and 4.6 respectively (where ductility is defined as the ratio of the total displacement to the yield displacement). Values of this magnitude can be achieved readily by mild steel components in tension. The loading state of the frame when the bottom mass displacements are of the order of 8 ins can conveniently be determined using the '2-D STRUCTURE' program (Appendix IV, L.P.1) and confirmation that the residual members in the bottom bay are still within the elastic range may be obtained. Based on a yield stress of 19.4 T/in^2 (Appendix III) the combined bending and axial stresses in the bottom columns (the critical members) will not exceed the elastic range until a lateral displacement of the bottom mass of 10.2 inches is reached.

Although the displacement/time plot of Figure 6.10 and other subsequent figures indicate that some shift of the zero amplitude

point of the elastic vibrations occurs towards the end of the plot, it does not follow that the water tank has developed a permanent set to one side. Providing that the stresses in the columns and horizontal struts do not exceed the elastic limits, the residual elastic system will ensure that the structure resumes its original position when the effects of the seismic disturbance have decayed away. An exception to this behaviour would occur if the initial balance between the prestressed members in any one bay is disturbed during the shaking so that the residual prestressing loads do not counterbalance each other. In such circumstances some permanent sway displacement would be expected after the movements resulting from the earthquake excitation have ceased.

The multiphase response of a half full tank (without any allowance for the convective action of the water) in similar conditions to the situation giving the results presented in Figure 6.9, is shown in Figure 6.11. Since the system barely exceeds the elastic range (confirmed by the very small calculated yield displacements) the maximum mass displacements are very similar in magnitude and time of occurrence to those determined for the extended elastic case (Figure 6.9).

In Figures 6.12 and 6.13 the full and half full tank extended elastic 2% damped responses - allowing for convective water action - of the elevated Moran and Cheney water tower, braced with rods of the sizes used in the original structure with no prestress and excited by the El Centro N/S accelerogram, are shown. The comparisons between the extended elastic displacements determined both neglecting

the convective action and including it, are summarised in Table 5.

Water State	Tank Full or Half full	Max. Tower Top Displacement (ins)	Time at Maximum Displacement (secs)
STATIC	1.0	4.644	14.58
STATIC	0.5	6.198	4.42
CONVECTIVE	1.0	8.561	17.91
CONVECTIVE	0.5	5.477	13.66

Table 5.

The full tank comparison emphasizes the possible amplification of the response which the shaking of the liquid may cause and supports the requirement[7] of a substantially increased seismic design coefficient for elevated fluid containers relative to the basic value specified for normal building structures.

Since the hypothetical extended elastic bracing invoked in the computations leading to the plots of Figures 6.5, 6.6, 6.7, 6.8, 6.9, 6.12, and 6.13 cannot, in fact, be achieved by typical mild steel braces of the dimensions specified, the actual displacement/time history of the Moran and Cheney frame - having prestressed braces and supporting a full water tank acting convectively - is of the form presented in Figure 6.14 in which the system responds in much the same manner as that described above when commenting on plot 6.10, namely it passes through a phase of acting as if there is no

effective bracing in the bottom bay between 6 seconds and 12 seconds, followed by a pattern of behaviour consistent with only one brace being active intermittently in the bottom bay. The ductility requirements of 2.9 and 2.5 are lower than those necessary for the non-convective water case and are well within the range available from typical mild steel elements.

The response of an identical system to the El Centro record factored by 0.60, serves to confirm that whereas in the extended elastic cases factoring of the excitation produces a direct linear variation in the responses (Figures 6.7 and 6.8) this relationship is not valid for the yielding systems. An increase from 0.60 to 1.0 in the excitation accelerogram amplitude only increases the displacement response amplitudes by about 30% at the critical lower mass level.

The introduction of bracing prestress in the earlier analyses was made in recognition of the fact that many tank structures do have the cross bracing pretensioned to ensure that it is effective in contributing to the lateral stiffness, in fact doubling it at low amplitudes of vibration until the prestress is cancelled by the effect of the lateral displacement. The extreme sensitivity of the system to the presence of pretension in the bracing is clearly indicated by a comparison of Figure 6.14 with Figure 6.16, which corresponds to the identical situation to that used to produce 6.14 except that no prestress was introduced into the braces. The absence of prestress results in the initially more flexible system taking longer to build up its maximum response (14.53 seconds instead

of 4.398 seconds) and the plot follows the extended elastic pattern of Figure 6.12 for the first 13 seconds after which, during the following 6 or 7 seconds, the effectively unbraced bottom bay breaks up the regular form of oscillation and brings about a substantial reduction in the vibrational amplitude in the later part of the plot. In this phase the bottom bay once more responds as a partially braced panel for some of the time. The bottom bay bracing ductility requirements are actually higher in the pretensioned case, having values of 3.8 and 1.4 in the system corresponding to the plot of Figure 6.16.

When a deliberate attempt is made to promote the development of multiphase response with the consequent break-up of the regular oscillatory pattern of the seismically excited structural system, plots of the form of those shown in Figures 6.17 and 6.18 may be obtained. In these the El Centro N/S accelerogram is used as the excitation applied to a Moran and Cheney frame with original (but prestressed) bracing in the top two bays and with modified prestressed bracing in the bottom bay. Cross bracing with each member having the equivalent of 2 ins^2 of high tensile steel in parallel with 2.82 ins^2 of mild steel is included in the bottom bay so that the total bracing steel area is equivalent to that originally provided. The 2% damped response of a full tank is shown in Figure 6.17 and that of a half full tank in Figure 6.18, in both cases provision for the convective action of the water is made. For each tank condition typed output from the XBFRME program confirms that the yielding of the mild steel and early fracturing of the sacrificial high tensile steel occurs within the first 3 seconds and the irregular pattern of the subsequent

response may be followed conveniently from the figures. The maximum ductilities of 4.7 and 5.5 can be readily provided by mild steel braces of the configuration envisaged and the maximum frame displacements are within the elastic range so that no yielding of the column or strut members would be expected to occur as a result of the seismic loading.

The analyses leading to the displacement/time plots presented in Figures 6.19, 6.20, 6.21 and 6.22 were undertaken to indicate the type of optimisation of the choice of bracing which may be undertaken using the XBFRME program. On the assumption that the design condition is selected as being the response to the El Centro N/S accelerogram of a full tank system (allowing for the convective action of the water) and that owing to the necessity to resist wind and other service loads satisfactorily, an initial value of lateral stiffness close to that provided by the Moran and Cheney frame with the original unprestressed bracing is required, several combinations of mild steel and high tensile steel braces were investigated. The multiphase responses presented in Figures 6.19, 6.20, 6.21 and 6.22 all involve the progressive yielding or fracturing of sacrificial prestressed braces. In each case the bottom bay mild steel braces yield and the corresponding high tensile steel braces fracture and in the cases corresponding to Figures 6.19, 6.20 and 6.21 the middle bay high tensile steel breaks also. Of the particular systems checked that corresponding to Figure 6.20 has both the smallest lateral displacement response and minimum ductility requirement and so could be selected in preference to the others. This example comprises a very

limited use of the bracing selection procedure made possible by the use of the XBFRME program but should be sufficient to illustrate the range of possible application.

VII.2 Considerations of Dissipated Energy as a Proportion of the Total Seismic Input Energy

Housner has suggested[24] that the maximum energy E attained by an oscillator subject to seismic excitation may be estimated using the expression

$$E = \tfrac{1}{2} m S_{v,n}^2$$

where m is the mass of the oscillator and $S_{v,n}$ is the velocity spectrum ordinate for damping n, assuming the velocity spectrum to consist of a straight horizontal line.

For the El Centro N/S component, at 2% critical damping $S_{v,0.02}$ may be taken to be equal to 2.05 ft/second[24] and hence the total energy input to the Moran and Cheney frame, supporting a full tank of water, resulting from this excitation will be approximately equal to

$$\tfrac{1}{2} \frac{952,600}{32.2} (2.05)^2 = 62,000 \text{ ft.lbs}$$

If this energy was entirely in the form of strain energy, the stress state in the members of the frame would correspond to a lateral load of

$$\frac{2\pi}{T} \cdot S_{v,n} = \frac{2\pi}{1.4} \cdot \frac{2.05}{32.2} \cdot g = 0.28g$$

where T, the period of oscillation, is taken as 1.4 second.

Moran and Cheney's estimate of 0.20g loading corresponding to the elastic limit of the frame would involve an elastic strain energy

in the structure of

$$\left(\frac{20}{28}\right)^2 \; 64{,}000 \;=\; 33{,}000 \text{ ft.lbs}$$

If a total energy input of 64,000 ft.lbs is to be absorbed, then that portion dissipated in plastic deformation may be estimated to be

$$62{,}000 - 33{,}000 \;=\; \underline{29{,}000} \text{ ft.lbs}$$

In the system giving rise to the displacement/time response shown in Figure 6.10, bottom bay bracing yields of 6.058 ins and 4.931 ins were calculated. The dissipated work corresponding to these plastic distortions may be calculated viz.-

$$\tfrac{1}{2} \times \frac{11{,}000}{12} \times 6.058^2 \;=\; 16{,}700$$

$$\tfrac{1}{2} \times \frac{11{,}000}{12} \times 4.931^2 \;=\; \underline{11{,}100}$$

$$\underline{27{,}800} \text{ ft.lb}$$

Allowing for the assumptions made by Housner, (which tend to over-estimate the plastic energy absorption requirement) and the replacement of the actual post yield axial stiffness curve of the bracing rods by a straight line (Figure 6.2) for the purpose of establishing a value for use in the XBFRME program (which procedure underestimates the plastic energy dissipation for small yield displacements) the correlation between the 29,000 ft.lbs and 27,800 ft.lbs is satisfactory.

In the case of allowance for convective water action being made - in the system having the response presented in Figure 6.14 - the dissipated work corresponding to the plastic displacements of 3.096 ins and 2.697 ins is only 7,700 ft.lb and the correlation

between the estimate of the requirement based on Housner's method and the value computed from the XBFRME results is much less close. However this result is consistent with the assumptions used in the Housner and XBFRME approaches.

If only that part of the water mass which moves with the tower is included in the total energy input calculation, the value becomes

$$\frac{1}{2}\,\frac{710,600}{32.2}\,(2.05)^2 \;=\; 46,000 \text{ ft.lbs}$$

Hence the dissipated energy requirement estimated using Housner's method is

$$46,000 - 33,000 \;=\; 13,000 \text{ ft.lb}$$

which correlates more satisfactorily with the value based on the XBFRME plastic deformations i.e. 7,700 ft.lb.

Certainly a calculation following the Housner approach will serve as a valuable check on the XBFRME yield distortion predictions, particularly in the non-convective water situations.

VII.3 The Choice of Accelerogram

Since the El Centro 1940 N/S accelerogram represented, until quite recently, the largest strong motion seismic excitation reliably recorded it has been used in almost all of the numerical integration studies made of seismic response. Justifiable criticism of the application of a particular earthquake record to a general response prediction procedure have been made and these have become more frequent as appreciation has grown of the sensitivity of a structure's response to the characteristics of the excitation used and, more recently, of the significant effects which local site conditions may have in

modifying the base rock seismic disturbance and consequently producing an excitation peculiar to the particular site on which a projected structure is to be built.

The El Centro 1940 N/S accelerogram was used exclusively in the analyses reported in Chapter Six in order to provide a common basis for comparison with previous studies and to eliminate a further variable in the sensitivity analyses. However the XBFRME program can equally well accept input from any of the Berg type records[70], six of which are available, representing a range of natural earthquakes differing in their spectural properties according to the characteristics of the shaking at the recording site.

In an attempt to overcome the problem of lack of generality posed by the use of a particular accelerogram record from a past earthquake Jennings et al[71] have produced simulated earthquake motions suitable for use in numerical integration analyses and these are freely available on request from the originators.

The Jennings type synthetic digitised records are designed to represent four types of earthquakes; the shaking near the causative fault in a large (Richter Magnitude > 8) earthquake, the shaking close to the fault in a Magnitude 7 earthquake, the motion expected in the epicentral region of a Magnitude 5.5 to 6 shock and the shaking close to the fault in a very small Magnitude 4.5 to 5.5 earthquake. In their original form these records do not conform to the Berg type format but suitable modifications have been made to produce card decks containing the digitised card decks in similar configuration to the Berg records. Consequently the XBFRME program can be run using any of

these synthetic earthquake records as the exciting accelerogram.

General acceptance of these synthetic accelerograms as the basis of seismic design analyses may have to wait until suitable techniques for allowing for the particular site conditions are more widely appreciated and for a better understanding of the dynamic properties of soils to be developed. In the meantime the use of records such as the El Centro accelerogram in numerical integration analyses is likely to continue. It can be no less valid as a design aid than the response curves included in current design codes[9] which are, in many cases, based on the El Centro and other natural earthquake records.

VII.4 Torsional Response Consideration

When a vertical axis through the centre of mass does not coincide with a similar vertical axis through the centre of resistance at each mass level, lateral seismic excitations at the base of a structure necessarily induce torsional oscillations in addition to the lateral vibration. That torsional displacements have occurred in elevated water towers has been reported[3, 20], consequently the possibility of torsional response must be considered at the design stage of elevated water tanks.

Because of the inherent simplicity of a cross braced elevated water tower it should be possible for the designer to maintain sufficient control of the structural configuration to ensure that the representation of the system as a plane frame is sufficiently realistic for design purposes, in which case the XBFRME program is applicable to the prediction of the response. However there may be

circumstances in which, due to the necessity to introduce off-centre inlet and outlet pipes or to link with adjacent structures, it does not prove possible to retain the symmetry of the system and in these circumstances the analysis technique must take account of the possible contribution of torsional displacement to the overall response. In principle the allowance may be made readily. In practice, because of the problems of accurately determining the mass and torsional stiffness distributions, it may be difficult to generate a sufficiently valid representation of the system to justify a substantial measure of confidence in the predictions made and it is suggested that a designer should make the avoidance of torsional response of prime importance when selecting the configuration of an elevated water tank structure. However, in those cases where this cannot be achieved, the response prediction technique presented in this thesis may be extended to the torsional response problem in the following manner.

An expression of the form of equation 4(1) can be written viz:-

$$\begin{bmatrix} M_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & M_N & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & M_1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & M_N & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & I_1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & I_N \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \cdot \\ \ddot{x}_N \\ \ddot{y}_1 \\ \cdot \\ \ddot{y}_N \\ \ddot{\theta}_1 \\ \cdot \\ \ddot{\theta}_N \end{Bmatrix} + \left[C\right]_T \begin{Bmatrix} \dot{x}_1 \\ \cdot \\ \dot{x}_N \\ \dot{y}_1 \\ \cdot \\ \dot{y}_N \\ \dot{\theta}_1 \\ \cdot \\ \dot{\theta}_N \end{Bmatrix} + \left[K\right]_T \begin{Bmatrix} x_1 \\ \cdot \\ x_N \\ y_1 \\ \cdot \\ y_N \\ \theta_1 \\ \cdot \\ \theta_N \end{Bmatrix} = -\left[M\right] \ddot{x}_g$$

in which x, y and θ are the two translational and the rotational displacements of each mass M

I_1 to I_N are the rotational inertias of the masses M_1 to M_N about the centre of gravity

and $\left[C\right]_T$ and $\left[K\right]_T$ are respectively the damping and stiffness matrix[73] for coupled translational and rotational response.

The techniques described in Chapter Four could be applied to the solution of multiphase response, suitable modifications being made to the coupled stiffness matrix as the bracing rods slacken, yield or fracture. In a situation in which the analysis would not be restricted by the limitations of the digital computer on which the study described in this thesis was undertaken there appears to be no obstacle to the concepts used in the XBFRME program being extended to torsional response prediction.

VII.5 Conclusion

Because of the interactive nature of the problem, seismic design procedures necessarily involve a cyclic approach to the determination of an acceptable structure. Until the configuration of the system is known the response cannot be predicted, but a knowledge of the response is needed if an adequate structural system is to be selected. The designer normally tackles the problem by basing an initial selection of structural configuration and member sizes on an equivalent static load, applying perhaps 0.1g or 0.2g lateral acceleration to all the masses, and then providing a structure which will satisfactorily

resist the combined dead and equivalent seismic loads without exceeding the elastic behaviour range. The determination of the response of this system by a dynamic analysis subsequently enables the expected deformations and member loads arising from the selected design earthquake to be calculated. If the stresses are below the acceptable limits and consistent with an economic use of material, the design is considered to be satisfactory. If these conditions are not met, a modified structure is selected and a further response prediction is undertaken. This cycle is repeated as often as required to achieve an acceptable design.

Techniques suitable for the prediction of the dynamic response in both the elastic[6] and post-yield[53,54] ranges of beam and column frame structures have been published previously although to date the post-yield analyses have been restricted to ideal elasto-plastic considerations. Attempts are currently being made[74] to extend the numerical integration approach to the more general case of degrading stiffness systems to represent more accurately the observed behaviour of reinforced and prestressed concrete frameworks in particular.

The method which is presented in this thesis of determining the seismic response of braced water tower structures in the yield range of the bracing has not been advanced previously. If it is to be of practical value a designer requires reassurance that specific multiphase response can be achieved reliably, without recourse to techniques which would prove impracticable with respect to detailing, fabrication or erection. It is submitted that the testing described in Chapter Three indicates that the provision of multiphase bracing is a

practicable proposition and that preselected behaviour can be achieved satisfactorily.

In so far as it has been possible to verify the mathematical modelling of an elevated fluid container by the small amplitude shaking tests described in Chapter Two and by correlation with both observed behaviour[21] and other predictions[24] this has been done. Further tests on prototype structures or large scale models, preferably involving excitation of the system beyond the elastic range of the bracing rods, are highly desirable and will undoubtedly be undertaken when facilities such as the earthquake simulator shaking table planned for the Richmond field station at the University of California at Berkeley become available.

In its present form the XBFRME program allows for the inclusion of one mild steel rod in each diagonal brace of a moment resistant frame. The program may be applied equally conveniently to pin jointed tower frameworks such as it would be required to analyse if a bolted or riveted non-moment-resistant frame was used to support a water tank. In such a situation if any residual elastic resistance was to be incorporated in the structure it would be essential to provide a bracing member, which would remain elastic, in parallel with the sacrificial one, in each case. Relatively little programming effort would be needed to extend the XBFRME program to allow for more than two parallel braces if this should be required. The number of storeys which may be catered for by the XBFRME program in the form presented in Chapter Five is limited by the output statements. For convenience these specifically apply to the three and four mass systems

analysed as described in Chapter Six. Minor modifications to the output prescriptions would allow extension to as many masses as may be required for a particular analysis, subject only to the storage limitations of the computer used.

If the design of elevated water tank structures is to be placed on a more rational basis than hitherto, it is evident that the operating conditions and the level of seismic resistance sought will each have to be specified more accurately than has been the case in the past. For instance, if a tank is to be used to provide a constant head supply it is most unlikely that it will be in any state other than either full or completely empty when excited by an earthquake. Hence it would not be necessary to check the response of a series of partially full conditions, as would be the case for a supply tank which could well be at any stage of draw down when an earthquake occurs. Where the possibility exists of the tank contents being frozen the situation in which the non-convective response could prove to be critical must be examined. The statistical probability of ground shaking of a particular magnitude being experienced at a chosen site should be taken into account in selecting a suitable exciting accelerogram and, wherever possible, the characteristics of this record should reflect the site properties. An assessment of the acceptability of sacrificial bracing elements must be made and, when such response is anticipated, suitable remedial action must be arranged to ensure speedy restoration of the structure after a major shock has occurred. Generally a more sophisticated approach than that used up to now will be necessary. The use of a

torsion spanner to set bracing prestress levels may appear to be an unfamiliar technique in civil engineering but the example set by the Japanese, who have showed their willingness to adopt elaborate detailing and to include ingenious devices in seismic resistant structures[75], could well be followed by others.

It is submitted that the method of predicting the seismic response of cross braced elevated water tower structures which is presented in this thesis is valid and that the use of this approach will enable designers to achieve earthquake resistance in this type of structure with greater confidence.

APPENDIX I

CALCULATIONS UNDERTAKEN IN MODELLING OF ILAM WATER TOWER

(a) Determination of Lateral Stiffness $\frac{3EI}{L^3}$

Second Moment of Area I for main column

$$\frac{\pi}{64}(d_1^4 - d_2^4) = \frac{\pi}{64}(57^4 - 54^4) = 101{,}000 \text{ in}^4$$

and for inner tubes

$$\frac{\pi}{64}(39^4 - 36^4) = 31{,}100 \text{ in}^4$$

$$\frac{\pi}{64}(25.5^4 - 23^4) = 7{,}500 \text{ in}^4$$

Hence, Total I = 140,000 in^4.

Taking Elastic Modulus E = 6×10^6 psi

and Length L = 360 inches

Lateral Stiffness k_{so} = 54,000 lb/in or 650,000 lb/ft

(b) Determination of Masses m and m_s

Bowl Contents m

Tank Capacity $\pi \times 4^2 \times 6.5 = 328 \text{ ft}^3$

Contents' Weight = $328 \times 62.4 = 20{,}500$ lbs

and $m = \frac{20{,}500}{g} = \frac{20{,}500}{32.2} = \underline{\underline{638}}$ lbs/g

Structure m_s

Bowl Concrete Volume $\frac{8 \times \pi \times 6.5}{4} \triangleq 40 \text{ ft}^3$

Bowl Weight $40 \times 150 = 6{,}000$ lbs

Insert Weight = 1530 lbs

$\therefore$ Total Weight = $\underline{7,530}$ lbs

Pipe Weights

4.5 ft I.D. pipe	$\frac{\pi \times 4.5 \times 150}{4}$	$\simeq$	500 lb/ft run
3.0 ft I.D. pipe	$\frac{\pi \times 3 \times 5 \times 150}{24}$	$\simeq$	300 lb/ft run
2.0 ft I.D. pipe		$\simeq$	200 lb/ft run
		Total	1000 lb/ft run

Thus Bowl Weight + $\frac{1}{3}$ Tower Weight = 7,530 + 10,000

= 17,530 lbs

in which case $m_s = \frac{17,530}{32.2}$ = $\underline{545}$ lbs/g

or, considering all pipes to be full of water,

Weight of Water = $\pi \times 2^2 \times 30 \times 62.4$ = 24,500 lbs

in which case $m_s = (17,530 + \frac{24,500}{3})$ = $\underline{800}$ lbs/g

Input to MODANAL Program, therefore,

k_{so} = 650,000

m = 0 → 638 since, when water height, m

0	0
2.0	196
3.5	340
5.0	490
6.5	638

and m_s = 545 → 800

APPENDIX II

POST YIELD AXIAL STIFFNESS OF BRACING RODS

True stress is the ratio of the load to the cross-sectional area at any instant.

and true strain $= \int_{L_o}^{L} \frac{\sigma L}{L}$ where σL is change in length and L is specimen length

$= \log_e(\frac{L}{L_o})$ where L_o is original specimen length

$= \log_e(1+s)$ where s is the normal engineering strain.

Now the relation between true stress σ_i and true strain ϵ_i may be expressed

$$\sigma_i = K_o(\epsilon_i)^n$$

where K_o is a constant known as the strength coefficient and n is a constant known as the strain hardening coefficient.

The true force/true deflection curve may be determined using the true stress/true strain relationship for a bracing rod, in the following manner.

The true force $= \sigma_i \times \frac{A_o L_o}{L_i}$ where A_o is original rod cross-sectional area and L_i is rod length when true force is determined.

$$= \frac{A_o L_o}{L_i} \cdot K_o(\epsilon_i)^n$$

Using this expression, the post yield true force, true deflection curves may be determined for the range of bracing rods used.

APPENDIX III

GEOMETRICAL AND STRUCTURAL PROPERTIES OF TEST FRAME

Centreline dimensions of frame 45 in x 36 in

Hence diagonal length 57.63 in

and angle with horizontal 38° 37'

Thus, horizontal component of bracing force $F = 0.782\ F$

and vertical component of bracing force $F = 0.624F$

Cross Bracing Stiffness Considerations

(neglecting second order effects)

Lateral Load $= 0.782F = 0.782\ EA\ \frac{\delta L}{L}$

where E is elastic modulus of bracing

A is cross sectional area of bracing

δL is change in length of bracing

L is original length of bracing.

Thus Lateral Load $= \frac{0.782^2 EA}{L} \times$ lateral deflection

and Lateral Stiffness $= \frac{0.782^2 EA}{L}$

In the case of initial pretensioning of diametrically opposed diagonal braces, this stiffness is doubled.

Where pairs of braces are used the stiffness becomes

$$0.782^2\ \Sigma\frac{EA}{L}$$

Elastic Limit Sway (d_{yb}) of Bracing

$$d_{yb} = 0.782 \frac{L}{E} \times \text{bracing yield stress}$$

Unbraced Portal Stiffness Considerations

Analysing half of the rigid knee portal

From diagram A III.1

$$M_{BA} + P.L_{AB} = 0$$

$$M_{AB} = 0$$

$$M_{CB} = 0$$

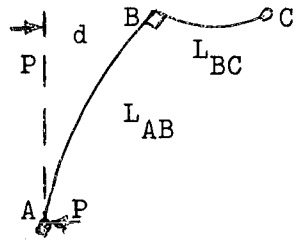

Fig. A III.1

Slope Deflection equations may be written

$$M_{AB} = \frac{2EI}{L_{AB}} (2\theta_A + \theta_B - \frac{3d}{L_{AB}})$$

$$M_{BA} = \frac{2EI}{L_{AB}} (2\theta_B + \theta_A - \frac{3d}{L_{AB}})$$

$$M_{BC} = \frac{2EI}{L_{BC}} (2\theta_B + \theta_C)$$

$$M_{CB} = \frac{2EI}{L_{BC}} (2\theta_C + \theta_B)$$

Since hinges exist at A and C

$$\theta_A = \tfrac{1}{2}(\frac{3d}{L_{AB}} - \theta_B)$$

$$\theta_C = -\frac{\theta_B}{2}$$

Equilibrium at B implies $M_{BA} + M_{BC} = 0$

$$\therefore L_{BC}(2\theta_B + \theta_A - \frac{3d}{L_{AB}}) + L_{AB}(2\theta_B + \theta_C) = 0$$

$$L_{BC}(2\theta_B + \frac{3d}{2L_{AB}} - \frac{\theta_B}{2} - \frac{3d}{L_{AB}}) + L_{AB}(2\theta_B - \frac{\theta_B}{2} = 0$$

$$L_{BC}\left(\frac{3}{2}\theta_B - \frac{3}{2}\frac{d}{L_{AB}}\right) + L_{AB}\left(\frac{3}{2}\theta_B\right) = 0$$

$$\therefore\ \theta_B = \frac{d}{L_{AB}}\frac{L_{BC}}{(L_{BC} + L_{AB})}$$

Hence

$$\frac{2EI}{L_{AB}}\left(\frac{3}{2}\frac{d}{L_{AB}}\frac{L_{BC}}{(L_{BC} + L_{AB})} - \frac{3}{2}\frac{d}{L_{AB}}\right) + P.L_{AB} = 0$$

and

$$\frac{P}{d} = \frac{-2EI}{L_{AB}^2}\left(\frac{3}{2} \cdot \frac{1}{L_{AB}} \cdot \frac{L_{BC}}{(L_{BC} + L_{AB})} - \frac{3}{2} \cdot \frac{1}{L_{AB}}\right)$$

Therefore Lateral Stiffness of Half of Rigid Knee Portal

$$= \frac{-2 \times 13{,}000 \times 2}{36^2}\left(\frac{3}{2}\frac{22.5}{36 \times 58.5} - \frac{3}{2}\frac{1}{36}\right)$$

$$= \frac{13{,}000}{18^2}(0.0417 - 0.0163) = \frac{13{,}000 \times 0.0254}{18^2}$$

$$= 1.02 \text{ Ton/in.}$$

Or, Lateral Stiffness of Unbraced Rigid Knee Test Portal

$$= 2.04 \text{ Ton/in.}$$

Elastic Limit Sway (d_{yp}) of Portal

Taking the yield stress to be 20 Ton/in^2. (Experimental value 19.4 Ton/in^2 - section III.3)

$$d_{yp} = \frac{M_{BA}}{1.02 \times 36} = \frac{20 \times 2}{1.02 \times 36 \times 1.5} = 0.76 \text{ in.}$$

APPENDIX IV

LISTINGS OF UNIVERSITY OF CANTERBURY CIVIL ENGINEERING DEPARTMENT'S LIBRARY PROGRAMS USED IN THE ANALYSES OF CROSS BRACED FRAMEWORKS

```
C
C
C
C
C     ******************************************************************
C     *                                                                *
C     *              CIVIL ENGINEERING PROGRAM LIBRARY                 *
C     *                                                                *
C     *PROGRAM FOR STRUCTURAL ANALYSIS BY DIRECT STIFFNESS METHOD USING*
C     *          FINITE ELEMENT CONCEPTS IN PLANE PROBLEMS             *
C     *                                                                *
C     ******************************************************************
C
C
0001        COMMON/PARA/NN,MP,NUMEL,NUMNP,NFP,NSTOP,NFIX(99),R(198)
0002        DATA FLAX/4HSTOP/
0003        DATA FLAG/4HRUNS/
0004        NSTOP=0
C
0005      5 READ10,CHECK
0006     10 FORMAT(A4)
0007        IF(CHECK.EQ.FLAX) GO TO 20
0008        IF(CHECK.NE.FLAG) GO TO 5
0009        NOW=KLOCK(NT)
0010        CALL LOAD('PHASEA')
0011        CALL SETUP
0012        IF(NSTOP.NE.0) GO TO 15
0013        CALL LOAD('PHASEB')
0014        CALL DRSTIF
0015        CALL LOAD('PHASEC')
0016        CALL FORCE
0017     15 NEXT=KLOCK(NTT)
0018        NETT=NEXT-NOW
0019        PRINT17,NETT
0020     17 FORMAT(//23H TOTAL TIME IN SECONDS=I5 )
0021        GO TO 5
0022     20 PRINT 30
0023     30 FORMAT(1H1)
0024        RETURN
0025        END
```

LIST L.P. 1

```
FORTRAN IV    MODEL 44  PS      VERSION 3,  LEVEL 3   DATE  71048

0001        SUBROUTINE SETUP
C
C     ******************************************************************
C     SUBROUTINE TO READ,CHECK DATA,FORM ELEMENT PROPERTIES,READ LOADS
C     ******************************************************************
C
0002        COMMON/PARA/NN,MP,NUMEL,NUMNP,NFP,NSTOP,NFIX(99),R(198)
0003        DIMENSION XORD(99),YORD(99),HEAD(20),NP(160,3),AR(160),SI(160), IP
           1ERM(3),A(3),B(3),RR(3)
0004        EQUIVALENCE (XORD,R),(R(100),YORD),(HEAD,NP)
0005        DATA IPERM/2,3,1/
C
0006        REWIND 1
0007        NSTOP=0
0008        READ10,HEAD
0009     10 FORMAT(20A4)
0010        PRINT12,HEAD
0011     12 FORMAT(1H1,20A4)
0012        DO 5 I=1,99
0013        XORD(I)=0.0
0014        YORD(I)=0.0
0015      5 NFIX(I)=0
0016        DO 6 I=1,160
0017        SI(I)=0.0
0018        AR(I)=0.0
0019        DO6J=1,3
0020      6 NP(I,J)=0
C
0021        READ15,NUMNP,NUMEL,KP,EMOD,XU
0022     15 FORMAT(3I5,2F10.3)
0023        NFP=3
0024        IF(KP.EQ.2) NFP=2
0025        PRINT 17,NUMNP,NUMEL,NFP,EMOD,XU
0026     17 FORMAT(//24H NUMBER OF NODAL POINTS=,I4/
           1          24H NUMBER OF ELEMENTS     =,I4/
           2          24H NUMBER OF EQNS/NODE    =,I4/
           3          24H ELASTIC MODULUS        =,E15.3/
           4          24H POISSONS RATIO         =,E15.3)
0027        NN=NFP*NUMNP
0028        IF(NN.GT.198)NSTOP=1
0029        IF(NUMEL.GT.160)NSTOP=1
0030        IF(NSTOP.NE.0) PRINT18
0031     18 FORMAT(31H INITIAL PARAMETER ERROR,CANCEL )
0032        IF(NSTOP.NE.0) RETURN
C
C
C     READ STRUCTURAL DATA
C
0033        READ20,(L,XORD(L),YORD(L),NFIX(L),N=1,NUMNP)
```

```
      2C FORMAT(I1C,2F1C.3,I5)
         PRINT21
      21 FORMAT(31H1NCDE       XORC       YORC   NFIX )
         PRINT22,(I,XCRD(I),YCRD(I),NFIX(I),I=1,NUMNP)
      22 FORMAT(I5,2F1C.3,I6)
C
C     FINITE MEMBERS
C
      25 READ 26,(L,(NP(L,I),I=1,3),AR(L),SI(L),N=1,NUMEL)
      26 FORMAT(4I4,F8.3,F1C.3)
         PRINT27
      27 FORMAT(46H1  N    I    J    K         AREA,TH          M OF I    )
         PRINT28,(L,(NP(L,I),I=1,3),AR(L),SI(L),L=1,NUMEL)
      28 FORMAT(4I4,2F15.3)
C
C     CHECK BAND WIDTH
C
         MAXDIF=30-NFP
         IFLAG=0
         MM=0
         DO5ON=1,NUMEL
         IF(NP(N,3).NE.C) GO TC 45
         NFF=NP(N,1)-NP(N,2)
         NFF=NFF*NFP
         NDIF=IABS(NFF)
         IF(NDIF.EQ.0.OR.NDIF.GT.MAXDIF) IFLAG=1
         IF(NDIF.GT.MM) MM=NDIF
         GC TO 48
      45 DO47I=1,3
         J=IPERM(I)
         NFF=NP(N,I)-NP(N,J)
         NFF=NFF*NFP
         NDIF=IABS(NFF)
         IF(NDIF.EQ.0.OR.NDIF.GT.MAXDIF) IFLAG=1
      47 IF(NDIF.GT.MM) MM=NDIF
      48 IF(IFLAG.NE.0) NSTCP=1
         IF(IFLAG.NE.C) PRINT42,N
      42 FORMAT(26H NCDE ARRAY ERRCR,MEMBER=,I5)
      5C IFLAG=0
         MM=MM+NFP
         IF(NSTOP.NE.0) RETURN
C
C     FORM ELEMENT PROPERTIES
C
         DO 100 N=1,NUMEL
         IF(NP(N,3).NE.C) GC TC 61
         I=NP(N,1)
         J=NP(N,2)
         K=C
         AA=XORD(J)-XCRD(I)
         BB=YORD(J)-YCRD(I)
         AREA=AR(N)
         SII=SI(N)
         IF(NFP.EQ.2) SII=C.0
         CALL BEAM (AA,BB,AREA,SII,EMCD,I,J)
         GO TC 1C0
      61 DO 7C I=1,3
         J=IPERM(I)
         K=IPERM(J)
         L=NP(N,J)
         M=NP(N,K)
         A(I)=XORD(M)-XCRD(L)
      7C B(I)=YORD(L)-YCRD(M)
         I=NP(N,1)
         J=NP(N,2)
         K=NP(N,3)
         THICK=AR(N)
         CALL SCST(A,B,THICK,I,J,K,EMCD,XU)
      8C CONTINUE
     1CC CONTINUE
C
C     READ LOADS
C
         PRINT105
     1C5 FORMAT(35H1NCDE      XLOAC      YLCAC      MCMENT )
         DO 110 N=1,198
     11C R(N)=0.C
         DO2CON=1,NUMNP
         READ120,M,(RR(I),I=1,NFP)
         IF(M.EQ.C) GC TO 201
     12C FORMAT(I5,3F10.3)
         PRINT120,M,(RR(I),I=1,NFP)
         L=(M-1)*NFP
         DO130K=1,NFP
         J=L+K
     13C R(J)=R(J)+RR(K)
     20C CONTINUE
    2C1  CONTINUE
C
         RE*IND 1
         RETURN
         END
```

LIST L.P 1 (CONTINUED)

```
0001            SUBROUTINE BEAM (XLX,XLY,AREA,SII,EMOD,II,JJ)
                C
                C     ******************************************************************
                C     BEAM MEMBER, PRISMATIC
                C     ******************************************************************
                C
0002            DIMENSION ST(6,6),TM(3,6),A(3,6)
                C
0003            KK=0
0004            COF=0.5
0005            XL=SQRT(XLX**2+XLY**2)
0006            S2=-XLY/XL
0007            C2=XLX/XL
0008            TEMP=4.0/XL*EMOD
0009            DO160I=1,3
0010            DO160J=1,3
0011       160  ST(I,J)=0.0
0012        5C  ST(1,1)=AREA/4.0*TEMP
0013            ST(2,2)=SII*TEMP
0014            ST(3,3)=ST(2,2)
0015            ST(2,3)=ST(2,2)*COF
0016            ST(3,2)=ST(2,3)
0017            TM(1,1)=-C2
0018            TM(1,2)=+S2
0019            TM(1,3)=0.0
0020            TM(2,1)=-S2/XL
0021            TM(3,1)=TM(2,1)
0022            TM(2,2)=-C2/XL
0023            TM(3,2)=TM(2,2)
0024            TM(2,3)=-1.0
0025            TM(3,3)=0.0
0026            DO 180 I=1,3
0027            TM(1,I+3)=-TM(1,I)
0028            TM(3,I+3)=-TM(2,I)
0029       180  TM(2,I+3)=-TM(3,I)
0030            TM(2,6)=0.0
0031            TM(3,6)=-1.0
0032            DO220I=1,3
0033            DO220J=1,6
0034            A(I,J)=0.0
0035            DO220K=1,3
0036       22C  A(I,J)=A(I,J)+ST(I,K)*TM(K,J)
0037            DO240I=1,6
0038            DO240J=1,6
0039            ST(I,J)=0.C
0040            DO240K=1,3
0041       240  ST(I,J)=ST(I,J)+TM(K,I)*A(K,J)
0042            WRITE (1) II,JJ,KK,ST,A
0043            RETURN
```

```
0001            SUBROUTINE SCST (A,B,THICK,II,JJ,KK,EMCD,XU)
                C
                C     ******************************************************************
                C     CONSTANT STRAIN TRIANGLE
                C     ******************************************************************
                C
0002            DIMENSION A(3),B(3),ST(6,6),S(3,6)
                C
0003            AREA=A(3)*B(2)-A(2)*B(3)
0004            C1=EMOD*THICK/(2.0*AREA*(1.0-XU**2))
0005            C2=C1*XU
0006            C3=0.5*C1*(1.0-XU)
0007            DO200I=1,3
0008            K=I+I
0009            DO200J=I,3
0010            L=J+J
0011            AA=A(I)*A(J)
0012            BB=B(I)*B(J)
0013            AB=A(I)*B(J)
0014            BA=B(I)*A(J)
0015            ST(K-1,L-1)=C1*BB+C3*AA
0016            ST(K,L)=C1*AA+C3*BB
0017            ST(K-1,L)=C2*BA+C3*AB
0018       20C  ST(K,L-1)=C2*AB+C3*BA
0019            DO300I=2,6
0020            DO300J=1,I
0021       300  ST(I,J)=ST(J,I)
0022            C1=C1*2.0
0023            C2=C2*2.C
0024            C3=C3*2.0
0025            DO400I=1,3
0026            K=I+I-1
0027            J=I+I
0028            S(1,K)=B(I)*C1
0029            S(1,J)=A(I)*C2
0030            S(2,K)=B(I)*C2
0031            S(2,J)=A(I)*C1
0032            S(3,K)=A(I)*C3
0033       4C0  S(3,J)=B(I)*C3
0034            WRITE (1) II,JJ,KK,ST,S
0035            RETURN
0036            END
```

LIST L.P.1 (CONTINUED)

```
C0C1            SUBROUTINE DRSTIF
                C
                C     ******************************************************************
                C     SUBROUTINE TO FORM TOTAL STIFFNESS MATRIX BY THE DIRECT STIFFNESS
                C     METHOD
                C     ******************************************************************
                C
C0C2            COMMON/PARA/NN,MM,NEL,NPT,NFP,NSTOP,NFIX(99),R(198)
C0C3            DIMENSION A(198,3C),R(198),ST(6,6),NP(3),S(3,6)
C0C4            EQUIVALENCE (R,B)
                C
C0C5            REWIND 1
CCC6            DO10I=1,NN
C0C7            DO10J=1,MM
C0C8          1C A(I,J)=0.C
                C
                C     SET UP TOTAL STIFFNESS MATRIX
                C
C0C9            DO100N=1,NEL
C010            READ(1) II,JJ,KK,ST,S
C011            NP(1)=II
C012            NP(2)=JJ
C013            NP(3)=KK
                C
0014            NNF=2
0015            NNE=3
C016            NFF=NFP
0017            IF(NP(3).EQ.C) NNE=2
C018            IF(NP(3).EC.C) NNF=3
0019            IF(NP(3).NE.C) NFF=2
C020            DO90I=1,NNE
C021            K=NP(I)
C022            KK=NNF*(I-1)
0023            IK=NFP*(K-1)
0024            DO90J=1,NNE
C025            L=NP(J)
C026            IF(K.GT.L) GC TO 90
0027            LL=NNF*(J-1)
0028            JK=NFP*(L-K)+1
0029            DC 8C II=1,NFF
C030            IS=KK+II
C031            IA=IK+II
C032            JA=JK-II
C033            DC 80 JJ=1,NFF
C034            JS=LL+JJ
0035            JA=JA+1
0036            IF(JA.LE.C) GC TC 80
0037            A(IA,JA)=A(IA,JA)+ST(IS,JS)
0038          8C CONTINUE
```

```
FORTRAN IV    MODEL 44  PS       VERSION 3,  LEVEL 3   DATE  71046

0039            90 CONTINUE
0040           100 CONTINUE
                C
                C     APPLY BOUNDARY CONDITIONS
                C
0041                DO240N=1,NPT
0042                M=(N-1)*NFP
0043                IF(NFIX(N).EC.C) GC TC 240
0044                I=100
C045                IF(NFP.EQ.2) I=10
0046                DO235J=1,NFP
0047                IF(NFIX(N).LT.I) GC TO 235
0048                L=M+J
C049                DO 230 K=1,MM
0050                L1=L+1-K
0051                A(L,K)=0.C
0052                IF(L1.LE.0) GC TC 230
0053                A(L1,K)=0.0
0054           230  CONTINUE
0055                A(L,1)=1.0
C056                R(L)=0.0
0057                NFIX(N)=NFIX(N)-I
0058           235  I=I/10
0059           240  CONTINUE
                C
                C     SOLVE SYSTEM FCR DISPLACEMENTS
                C
0060                NA=NN
0061                NB=MM
0062                CALL SYMSOL(NA,NB,1,A,R)
0063                CALL SYMSCL(NA,NB,2,A,R)
                C
0064                REWIND 1
0065                RETURN
0066                END
```

LIST L.P. 1 (CONTINUED)

```
C0C1            SUBROUTINE SYMSOL(NN,MM,KKK,A,B)                        SYM4ECC1
                                                                        SYMS  C2
C               ********************************************************SYMS  C3
C               SUBROUTINE TO SOLVE BANDED SYMMETRIC MATRIX EQUATION AB=RSYMS  C4
C               ********************************************************SYMS  C5
C                                                                       SYMS  C6
C0C2            DIMENSION A(198,30),B(198)
C                                                                       SYMS  C8
C0C3            GO TO (1000,2000),KKK                                   SYMS  C9
C                                                                       SYMS  10
C               REDUCE MATRIX                                           SYMS  11
C                                                                       SYMS  12
C0C4         1000 DO 280N=1,NN                                          SYMS  13
C0C5            DO 260L=2,MM                                            SYMS  14
C0C6            C=A(N,L)/A(N,1)                                         SYMS  15
C0C7            I=N+L-1                                                 SYMS  16
C0C8            IF(NN.LT.I) GO TO 260                                   SYMS  17
C0C9            J=0                                                     SYMS  18
C0C10           DO 250K=L,MM                                            SYMS  19
C0C11           J=J+1                                                   SYMS  20
C0C12        250 A(I,J)=A(I,J)-C*A(N,K)                                 SYMS  21
C0C13        260 A(N,L)=C                                               SYMS  22
C0C14        280 CONTINUE                                               SYMS  23
C0C15           GO TO 500                                               SYMS  24
C                                                                       SYMS  25
C               REDUCE VECTOR                                           SYMS  26
C                                                                       SYMS  27
C0C16        2000 DO 290N=1,NN                                          SYMS  28
C0C17           DO 285L=2,MM                                            SYMS  29
C0C18           I=N+L-1                                                 SYMS  30
C0C19           IF(NN.LT.I) GO TO 290                                   SYMS  31
C0C20        285 B(I)=B(I)-A(N,L)*B(N)                                  SYMS  32
C0C21        290 B(N)=B(N)/A(N,1)                                       SYMS  33
C                                                                       SYMS  34
C               BACK SUBSTITUTION                                       SYMS  35
C                                                                       SYMS  36
C0C22           N=NN                                                    SYMS  37
C0C23        300 N=N-1                                                  SYMS  38
C0C24           IF(N.EQ.0) GO TO 500                                    SYMS  39
C0C25           DO 400K=2,MM                                            SYMS  40
C0C26           L=N+K-1                                                 SYMS  41
C0C27           IF(NN.LT.L) GO TO 400                                   SYMS  42
C0C28           B(N)=B(N)-A(N,K)*B(L)                                   SYMS  43
C0C29        400 CONTINUE                                               SYMS  44
C0C30           GO TO 300                                               SYMS  45
C                                                                       SYMS  46
C0C31        500 RETURN                                                 SYMS  47
C0C32           END                                                     SYMS  48
```

```
C0C1            SUBROUTINE FORCE
C               ********************************************************
C               FORCE OUTPLT
C               ********************************************************
C
C0C2            COMMON/PARA/NN,MM,NUMEL,NUMNP,NFP,NSTOP,NFIX(99),R(198)
C0C3            DIMENSION ST(3,6),SS(6,6),RB(6),S(3)
C0C4            DATA PONE/4HBEAM/
C0C5            DATA PTWO/4H CST/
C
C0C6            REWIND 1
C
C               PRINT DISPLACEMENTS
C
C0C7            IF(NFP.EQ.2) PRINT 260
C0C8            IF(NFP.EQ.3) PRINT 262
C0C9         260 FORMAT(35H1NODE X-DISPLACEMENT Y-DISPLACEMENT )
C0C10        262 FORMAT(50H1NODE X-DISPLACEMENT Y-DISPLACEMENT THETA-ROTATION )
C0C11           DO280N=1,NUMNP
C0C12           I=(N-1)*NFP+1
C0C13           IF(NFP.EQ.2) PRINT 270,N,R(I),R(I+1)
C0C14           IF(NFP.EQ.3) PRINT 272,N,R(I),R(I+1),R(I+2)
C0C15        270 FORMAT(I5,1PE15.4,1PE15.4)
C0C16        272 FORMAT(I5,1PE15.4,1PE15.4,1PE15.4)
C0C17        280 CONTINUE
C
C               PRINT ELEMENT FORCES.
C
C0C18           PRINT 290
C0C19           PRINT 291
C0C20        290 FORMAT( 67H1  N    I    J    K  BEAM     AXIAL FORCE     MOMENT AT I
                1 MOMENT AT J      )
C0C21        291 FORMAT( 67H                          CST        X-STRESS        Y-STRESS
                1    XY-STRESS      )
C0C22           DO400N=1,NUMEL
C0C23           READ (1) II,JJ,KK,SS,ST
C0C24           IF(KK.NE.0) GO TO 310
C0C25           K=(II-1)*NFP
C0C26           L=(JJ-1)*NFP
C0C27           DO 305 I=1,NFP
C0C28           J=K+I
C0C29           M=L+I
C0C30           RB(I)=R(J)
C0C31        305 RB(I+3)=R(M)
C0C32           IF(NFP.EQ.2)RB(3)=0.0
C0C33           IF(NFP.EQ.2)RB(6)=0.0
C0C34           DO 307 I=1,3
C0C35           S(I)=0.0
```

LIST L.P.1 (CONTINUED)

```
C036              DO 307 J=1,6
C037          307 S(I)=S(I)+ST(I,J)*RB(J)
C038              GO TO 320
C039           310 I=(II-1)*NFP+1
C040               J=(JJ-1)*NFP+1
C041               K=(KK-1)*NFP+1
C042               RB(1)=R(I)
C043               RB(2)=R(I+1)
C044               RB(3)=R(J)
C045               RB(4)=R(J+1)
C046               RB(5)=R(K)
C047               RB(6)=R(K+1)
C048               DO350I=1,3
C049               S(I)=0.0
C050               DO350J=1,6
C051          350 S(I)=S(I)+ST(I,J)*RB(J)
C052          320 CONTINUE
C053               IF(KK.EQ.0) PRINT 360,N,II,JJ,KK,PONE,(S(I),I=1,3)
C054               IF(KK.NE.0) PRINT 360,N,II,JJ,KK,PTWO,(S(I),I=1,3)
C055          360 FORMAT(4I4,2X,A4,1P3E15.5)
C056          400 CONTINUE
C057              REWIND 1
C058              RETURN
C059              END
```

INPUT DATA

ANALYSIS OF MORAN AND CHENGS WATER TOWER (WITH SYM BRACING)

NUMBER OF NODAL POINTS= 8
NUMBER OF ELEMENTS = 15
NUMBER OF EQNS/NODE = 3
ELASTIC MODULUS = 0.29CE 08
POISSONS RATIO = 0.C

NODE	XORD	YORD	NFIX
1	0.0	C.0	110
2	386.000	C.0	110
3	21.000	432.CCC	0
4	365.CCC	432.CCC	0
5	41.C00	851.C0C	0
6	345.000	851.C0C	0
7	61.000	1260.C0C	0
8	325.000	1260.000	0

N	I	J	K	AREA,TH	M OF I
1	1	3	0	30.600	1856.000
2	3	4	0	15.660	236.000
3	2	4	0	30.600	1856.C00
4	3	5	0	26.80C	1640.C00
5	5	6	0	14.520	173.600
6	4	6	0	26.80C	1640.C0C
7	5	7	0	22.0CC	1346.000
8	7	8	0	100.CCC	4000.C00
9	6	8	0	22.000	1346.000
10	1	4	0	4.820	0.0
11	2	3	0	4.820	0.0
12	3	6	0	7.100	0.0
13	4	5	0	7.10C	0.0
14	5	8	0	8.660	0.0
15	6	7	0	8.66C	0.0

NODE	XLOAD	YLOAD	MOMENT
3	0.500	C.C	0.C
4	0.500	C.C	0.C

OUTPUT

NODE	X-DISPLACEMENT	Y-DISPLACEMENT	THETA-ROTATION
1	-0.C	-0.C	-1.3593E-C8
2	-0.C	-0.C	-1.3593E-C8
3	4.6024E-C6	5.2914E-08	-4.6823E-C9
4	4.6024E-C6	-5.2913E-08	-4.6823E-C9
5	4.7658E-C6	4.6801F-09	7.1846E-10
6	4.7658E-C6	-4.6801E-09	7.1847E-10
7	4.8840E-C6	4.0668E-09	-3.6934E-10
8	4.8840E-C6	-4.0669E-09	-3.6935E-10

N	I	J	K	BEAM CST	AXIAL FORCE X-STRESS	MOMENT AT I Y-STRESS	MOMENT AT J XY-STRESS
1	1	3	0	BEAM	5.66929E-01	-9.53674E-07	-2.21775E 00
2	3	4	0	BEAM	2.86102E-06	5.22216E-01	5.22216E-01
3	2	4	0	BEAM	-5.66929E-01	-1.90735E-06	-2.21775E 00
4	3	5	0	BEAM	3.11613E-03	1.69552E 00	4.70949E-01
5	5	6	0	BEAM	2.86102E-06	-1.01983E-01	-1.01983E-01
6	4	6	0	BEAM	-3.11661E-03	1.69552E 00	4.70947E-01
7	5	7	0	BEAM	-5.46873E-04	-3.68874E-01	-1.61487E-01
8	7	8	0	BEAM	1.5258PE-05	1.61475E-01	1.61481E-01
9	6	8	0	BEAM	5.46515E-04	-3.68875E-01	-1.61485E-01
10	1	4	0	BEAM	7.2415CE-01	C.0	C.0
11	2	3	0	BEAM	-7.24149E-01	0.0	0.C
12	3	6	0	BEAM	4.18016E-03	0.0	0.C
13	4	5	0	BEAM	-8.17923E-03	0.0	C.C
14	5	8	0	BEAM	-2.22950E-03	0.0	0.C
15	6	7	0	BEAM	2.22935E-03	C.C	C.C

TOTAL TIME IN SECONDS= 20

LIST L.P. 1 (CONTINUED)

```
C
C
C     ******************************************************************
C     *                                                                *
C     *                CIVIL ENGINEERING PROGRAM LIBRARY               *
C     *                                                                *
C     *                   NORMAL MODES FROM F(LAT)                     *
C     *                                                                *
C     ******************************************************************
C
C
C     IL=1,OMEGA PRINTED., IL=0, NOT PRINTED
C     IM=1 PUNCHES IG DISPLACEMENTS, IM=0 SKIPS.
0001        DIMENSION A(30,30),W(38),X(30),X2(38),X3(30),S(30),D(30)
0002      1 READ30,NS,NM,E,CONV,IDAMP,IL,IM
0003     30 FORMAT(2I3,2E10.4,3I3)
0004        PRINT97,NS,NM,E,CONV,IDAMP,IL,IM
0005     97 FORMAT(1H 2I5,2E14.6,3I5)
0006        IL=IL+1
0007        IM=IM+1
0008        DO2I=1,NS
0009        DO2J=1,NS,5
0010      2 READ31,A(I,J),A(I,J+1),A(I,J+2),A(I,J+3),A(I,J+4)
0011     31 FORMAT(5L14.8)
C           READ DIAGONAL MASS MATRIX AND FORM PRODUCT (F)(M)
0012        DO29I=1,NS,8
0013     29 READ32,W(I),W(I+1),W(I+2),W(I+3),W(I+4),W(I+5),W(I+6),W(I+7)
0014     32 FORMAT(8E10.4)
0015        DO72I=1,NS
0016        S(I)=0.
0017     72 D(I)=0.
0018        J=(NS+4)*5/5
0019        L=NS+1
0020        DO90I=L,J
0021     90 X(I)=0.
0022        K=(NS+7)*8/8
0023        DO91I=L,K
0024     91 X2(I)=0.
0025        DO28J=1,NS
0026     28 W(J)=W(J)/386.4
0027        DO25I=1,NS
0028        DO25J=1,NS
0029     25 A(I,J)=A(I,J)*W(J)
C           DETERMINE LATENT VECTOR AND LATENT ROOT
0030        M=1
0031      7 DO26J=1,NS
0032     26 X(J)=1.0
0033        N=0
0034        P=0.
0035     46 DO27I=1,NS
0036        T=0.
0037        DO40J=1,NS
0038     40 T=T+A(I,J)*X(J)
0039     27 X2(I)=T
0040        WEXP2=1.0/X2(NS)
0041        DO41J=1,NS
0042     41 X(J)=X2(J)*WEXP2
0043        IF(P/WEXP2-1.0-CONV)42,42,43
0044     42 IF(P/WEXP2-1.0+CONV)43,44,44
0045     43 P=WEXP2
0046        N=N+1
0047        GO TO 46
0048     44 PRINT33,M,N
0049     33 FORMAT('1MODE=',I3,20X,'NUMBER OF CYCLES=',I3)
0050        T=0.
0051        DO9I=1,NS
0052        IF(ABS(T)-ABS(X(I)))8,9,9
0053      8 T=X(I)
0054      9 CONTINUE
0055        DO10I=1,NS
0056     10 X(I)=X(I)/T
0057        PRINT34,WEXP2,P
0058     34 FORMAT('0ANGULAR FREQUENCY SQUARED=',E14.8,5X,E14.8)
0059        GO TO (77,78),IL
0060     78 Q=SQRT(WEXP2)
0061        WRITE(7,372) Q
0062    372 FORMAT(1X,5E20.8)
0063     77 P=SQRT(WEXP2)/6.2831853
0064        Q=1.0/P
0065        PRINT35,P,Q
0066     35 FORMAT('0FREQUENCY=',E14.4,9X,'PERIOD=',E14.4)
0067        IF(IDAMP-2)73,74,75
0068     73 P=2.12
0069        T=2.10
0070        GO TO 76
0071     74 P=1.28
0072        T=1.26
0073        GO TO 76
0074     75 I=(IDAMP-5)*(40-IDAMP)/150
0075        P=0.8**I
0076        T=P-0.07
0077     76 IF(Q-9.55)66,67,67
0078     66 AM=0.37+(T-0.37)*(Q-1.43*Q*Q)/0.175
0079        GO TO 68
0080     67 AM=P*0.44/Q
0081     68 PRINT36,AM
0082     36 FORMAT('0AMPLIFICATION FACTOR=',F8.2)
```

LIST L.P. 2

```
C       DETERMINE IG DISPL. AND SHEAR FORCES.
0083        CONV=CONV+CONV
0084        P=0.
0085        DO531I=1,NS
0086     53 P=P+W(I)*X(I)
0087        T=0.
0088        DO541I=1,NS
0089     54 T=T+W(I)*X(I)*X(I)
0090        Q=P/T
0091        DO551I=1,NS
0092        X2(I)=386.4/WEXP2*Q*X(I)
0093     55 D(I)=D(I)+X2(I)*AM*X2(I)*AM
0094        DO561J=1,NS
0095        X3(J)=386.4*Q*W(J)*X(J)
0096     56 S(J)=S(J)+AM*AM*X3(J)*X3(J)
0097        GO TO (81,82),IM
0098     82 DO831I=1,NS,8
0099     83 PUNCH32,X2(I),X2(I+1),X2(I+2),X2(I+3),X2(I+4),X2(I+5),
          1X2(I+6),X2(I+7)
0100     81 PRINT39
0101     39 FORMAT('0DISPLACEMENT RATIOS.',6X,'1G DISPLACEMENTS.',10X,'1G FORC
          1ES.')
0102        J=NS
0103     59 PRINT38,X(J),X2(J),X3(J)
0104     38 FORMAT(7X,F8.4,14X,E10.4,14X,E10.4)
0105        IF(J-1)64,64,65
0106     65 J=J-1
0107        GO TO 59
C       MODIFY (F)(M) TO GIVE CONVERGENCE TO HIGHER MODES.
0108     64 DO581I=1,NS
0109     58 X(I)=X(I)/SQRT(T)
0110        IF(M-NM)47,69,69
0111     47 DO601I=1,NS
0112        DO60J=1,NS
0113     60 A(I,J)=A(I,J)-X(I)*X(J)*W(J)/WEXP2
0114        M=M+1
0115        GO TO 7
0116     69 PRINT37
0117     37 FORMAT('1',22X,'PREDICTED ELASTIC RESPONSE.'//'     DISPLACEMENT.'
          1,13X,'SHEARS.',17X,'FORCES.')
0118        P=0.
0119        J=NS
0120     71 S(J)=SQRT(S(J))
0121        D(J)=SQRT(D(J))
0122        X(J)=S(J)+P
0123        PRINT371,D(J),X(J),S(J)
0124    371 FORMAT(6X,E10.4,13X,E10.4,14X,E10.4)
0125        P=X(J)
0126        IF(J-1)1,1,70

0127     70 J=J-1
0128        GO TO 71
0129        END
```

OUTPUT

```
MODE=  1                    NUMBER OF CYCLES=  3
ANGULAR FREQUENCY SQUARED=0.30278134E 00      0.30278134E 00
FREQUENCY=     0.8758E-01          PERIOD=     0.1142E 02
AMPLIFICATION FACTOR=     0.03
DISPLACEMENT RATIOS.       1G DISPLACEMENTS.          1G FORCES.
        1.0000                 0.1282E 04                0.9355E 06
        0.8956                 0.1149E 04                0.1165E 05
        0.5380                 0.6899E 03                0.7400E 04


MODE=  2                    NUMBER OF CYCLES=  6
ANGULAR FREQUENCY SQUARED=0.11579932E 03      0.11579881E 03
FREQUENCY=     0.1713E 01          PERIOD=     0.5839E 00
AMPLIFICATION FACTOR=     0.60
DISPLACEMENT RATIOS.       1G DISPLACEMENTS.          1G FORCES.
       -0.0167                -.1988E-01                -.5547E 04
        0.7055                 0.8402E 00                0.3259E 04
        1.0000                 0.1191E 01                0.4896E 04


MODE=  3                    NUMBER OF CYCLES=  4
ANGULAR FREQUENCY SQUARED=0.92898401E 03      0.92900562E 03
FREQUENCY=     0.4851E 01          PERIOD=     0.2061E 00
AMPLIFICATION FACTOR=     0.67
DISPLACEMENT RATIOS.       1G DISPLACEMENTS.          1G FORCES.
       -0.0071                -.4517E-03                0.1011E 04
        1.0000                -.6341E-01                -.1973E 04
       -0.6751                 0.4281E-01                0.1409E 04


                        PREDICTED ELASTIC RESPONSE.

    DISPLACEMENT.              SHEARS.                   FORCES.
     0.3953E 02                0.2904E 05                0.2904E 05
     0.3541E 02                0.3143E 05                0.2394E 04
     0.2128E 02                0.3454E 05                0.3101E 04
   3    3   0.1000000E 01   0.1000000E-04   10    0     0
```

LIST L.P. 2 (CONTINUED)

```
C       CE020  CIVIL ENGINEERING PROGRAM LIBRARY  DAMPING MATRIX FORMATION
C
C      SIMPLIFIED CALC. OF (C) MATRIX                                        1
       DIMENSION S(30,30),C(30,30),W(30)                                     2
       INP=5                                                                 3
       LNP=6                                                                 4
       IPCH=7                                                                5
C      READ NO. OF STOREYS,PER CENT DAMPING 1ST AND 2ND MODES               51
       READ(INP,33) NS,C1,C2                                                 6
   33  FORMAT(I3,2F3.0)                                                      7
C      READ ANGULAR FREQUENCY 1ST MODE (RADIANS)                            71
       READ(INP,34) W1                                                       8
   34  FORMAT(5E14.8)                                                        9
C      READ ANGULAR FREQUENCY 2ND MODE (RADIANS)                            91
       READ(INP,34) W2                                                      10
       CG=.02*(C2*W2-C1*W1)/(W2*W2-W1*W1)                                   11
       CT=.02*C1*W1-CG*W1*W1                                                12
       WRITE(6,7) CG,CT
    7  FORMAT(2E14.7)
C      READ LATERAL STIFFNESS MATRIX                                       131
    1  READ(INP,94)((S(I,J),J=1,NS),I=1,NS)
   94  FORMAT(8E10.3)
C      READ LUMPED STOREY WEIGHTS.                                         141
       READ(INP,32) (W(I),I=1,NS)                                           15
   32  FORMAT(8E10.4)                                                       16
       DO 4 I=1,NS                                                          17
       DO 3 J=1,NS                                                          18
    3  C(I,J)=CG*S(I,J)                                                     19
    4  C(I,I)=C(I,I)+CT*W(I)/386.4                                          20
       WRITE(LNP,35) NS,C1,C2,W1,W2                                         21
   35  FORMAT(//1X,I3,2F5.1,2E13.3//)                                       22
       DO 2 I=1,NS                                                          23
    2  WRITE(LNP,36) (S(I,J),J=1,NS)                                        24
   36  FORMAT(1X,5E13.3)                                                    25
       WRITE(LNP,38)
       WRITE(LNP,37) (W(I),I=1,NS)                                          26
   37  FORMAT(1X,10E13.3)                                                   27
   38  FORMAT(//)                                                           28
       WRITE(LNP,38)                                                        29
       DO 6 I=1,NS                                                          30
       WRITE(LNP,36) (C(I,J),J=1,NS)                                        31
    6  WRITE(IPCH,39) (C(I,J),J=1,NS)                                       32
   39  FORMAT(5E14.7)                                                       33
       WRITE(LNP,40)                                                        34
   40  FORMAT(1X,'PROCESSING COMPLETE')                                     35
       STOP                                                                 36
       END                                                                  37
```

LIST L.P. 2A

```
C
C
C     ******************************************************************
C     *                                                                *
C     *          MATRIX PROGRAM....READ,PRINT,INVERT,PRINT.            *
C     *                                                                *
C     ******************************************************************
C
C
0001        DIMENSION A(10C,1C0)
C002      1 READ1CC,N
0003     10C FORMAT(I5)
C004        READ101,((A(I,J),I=1,N),J=1,N)
C005     101 FORMAT(8F10.0)
00C6        CALL PRINTM(A,N,1)
0007        CALL MATINV(A,N)
C0C8        CALL PRINTM(A,N,2)
C0C9        GO TC 1
C010        END
```

```
0001        SUBROUTINE MATINV(A,NMAX)
C           MATRIX INVERSION (A,NMAX)
0002        DIMENSION A(10C,1C0)
C003        DO 200 N=1,NMAX
C004        D=A(N,N)
0005        DO 100 J=1,NMAX
C006     10C A(N,J)=-A(N,J)/D
C007        DO 150 I=1,NMAX
0008        IF(N-I) 110,15C,110
0009     110 DO 140 J=1,NMAX
0010        IF(N-J) 120,14C,120
0011     12C A(I,J)=A(I,J)+A(I,N)*A(N,J)
0012     14C CONTINUE
0013     15C A(I,N)=A(I,N)/D
0014        A(N,N)=1.0/D
0015     20C CONTINUE
0016        RETURN
0017        END
```

LIST L.P. 3

```
FORTRAN IV    MODEL 44  PS       VERSION 3,  LEVEL 3   DATE  71048

 C001             SUBROUTINE PRINTM(A,N,L)
 C002             DIMENSION A(1CC,1C0)
 0003             WRITE(6,1C0) L
 00C4          10C FORMAT(///'0MATRIX NUMBER'I4/
                1       ' ------ ------ ---'//)
 C005             IF(N.GT.10) GO TC 2
 C006             DO 1 I=1,N
 0007           1 WRITE(6,1C1) (A(I,J),J=1,N)
 C008          101 FORMAT(' '10G13.3)
 C009             GO TO 5
 0010           2 DO 3 I=1,N
 0011           3 WRITE(6,101) (A(I,J),J=1,10)
 0012             WRITE(6,1C2)
 0013          102 FORMAT(//)
 0014             DO 4 I=1,N
 C015           4 WRITE(6,1C1) (A(I,J),J=11,N)
 0016           5 RETURN
 0017             ENC
```

OUTPUT

```
MATRIX NUMBER   1
------ ------ ---


    0.246E 06   -0.163E 06    0.292E 05
   -0.163E 06    C.306E 06   -0.144E 06
    0.292E 05   -C.144E 06    C.12CE C6


MATRIX NUMBER   2
------ ------ ---


    0.879E-05    C.857E-05    C.819E-05
    0.857E-05    C.16CE-04    C.171E-04
    0.819E-05    C.171E-04    C.27CE-04

 0A217I
/E
```

```
C
C     ******************************************************************
C     *                                                                *
C     *                CIVIL ENGINEERING PROGRAM LIBRARY               *  CCCCC1
C     *                                                                *
C     *                        PROGRAM  ELRES                          *  CCCCC2
C     *                                                                *
C     *   THIS PROGRAM DETERMINES THE  ELASTIC RESPONSE OF MULTI-MASS  *  CCCCC3
C     *         SYSTEMS TO BERG FORMAT EARTHQUAKE RECORDS.             *  CCCCC4
C     *                                                                *
C     ******************************************************************
C
C
C0001       CALL TITLE
C           MS=1 ENTER (K)LAT,MS=2 ENTER (F)LAT                          CCCCC5
C0002       DIMENSION S(20,20),F(20,20),QM(20),AA(20),BB(20),X(20),      CCCCC6
           1VEL(20),ACC(20),Y(20),Z(20),T(5),G(5),BX(20),TX(20),C(20,20) CCCCC7
C           N=NO OF MASSES,KP=NO OF STEPS BEFORE PRINTING,DT=STEP INTERVAL CCCCC8
C           TR=LENGTH OF RECORD                                          CCCCC9
0003       28 READ29,N,MS,KP,DT,TR                                       CCCC10
0004       29 FORMAT(3I3,2E10.4)                                         CCCC11
0005          PRINT321,N,MS,KP,DT,TR                                      CCCC1
C0006     321 FORMAT('1 INPUT DATA ECHO CHECK'/'0',3I5,2E13.4)
0007       32 FORMAT(3I4,2E13.4)                                         CCCC13
0008          PRINT34                                                     CCCC14
C0009      34 FORMAT(//)                                                 CCCC15
C0010         KC=0                                                       CCCC16
C0011      30 FORMAT(8E10.4)                                             CCCC17
C           READ (K)LAT OR (F)LAT                                        CCCC18
C0012         IF(MS.EQ.1)PRINT 51
C0013         IF(MS.EQ.2)PRINT 52
C0014      51 FORMAT('0',30X,'LATERAL STIFFNESS MATRIX'/)
C0015      52 FORMAT('0',30X,'LATERAL FLEXIBILITY MATRIX'/)
0016          DO40I=1,N                                                  CCCC19
C0017         READ31,(S(I,J),J=1,N)                                      CCCC20
0018       40 PRINT143,(S(I,J),J=1,N)                                    CCCC21
C0019     143 FORMAT(1X,10E13.4)                                         CCCC22
0020          PRINT34                                                    CCCC23
C           READ DAMPING MATRIX                                          CCCC24
0021          WRITE(6,53)
0022       53 FORMAT('0',40X,'DAMPING MATRIX'/)
0023          DO41I=1,N                                                  CCCC25
0024          READ31,(C(I,J),J=1,N)                                      CCCC26
0025       41 PRINT143,(C(I,J),J=1,N)                                    CCCC27
0026          PRINT34                                                    CCCC28
0027       31 FORMAT(5E14.8)                                             CCCC29
0028          WRITE(6,54)
0029       54 FORMAT('0',50X,'MASSES'/)
0030          GO TO (2,3),MS                                             CCCC30
0031        3 CALL MATINV(S,N)
C           READ MASS MATRIX
0032        2 READ30,(QM(I),I=1,N)
0033          PRINT143,(QM(I),I=1,N)
0034          WRITE(6,55)
0035       55 FORMAT('0',25X,50('**')//'0    TIME',30X,'DISPLACEMENTS OF MASSES'/
           1)
0036          DO5I=1,N
0037          X(I)=0.
0038          VEL(I)=0.
0039          BX(I)=0.
0040          QM(I)=QM(I)/386.4
0041          Z(I)=0.
0042          Y(I)=0.
0043          DO5J=1,N
0044        5 F(I,J)=0.
0045          IS=1
C           READ EQU. RECORD CARD
0046          READ33,ISC,(T(I),G(I),I=2,5)
0047          IF(IS-ISC)43,42,43
0048       33 FORMAT(I3,4(F8.4,F9.6))
0049       42 M2=2
0050          P=-G(2)*386.4
0051          DG=DT*386.4*(G(3)-G(2))/(T(3)-T(2))
0052          DO6I=1,N
0053        6 F(I,I)=1.0/QM(I)
0054          TM=0.
0055          DO7I=1,N
0056        7 Y(I)=QM(I)*P-Z(I)-Y(I)
0057          CALL VECMTP(ACC,F,Y,N,N)
0058          DO8I=1,N
0059          F(I,I)=QM(I)
0060          DO8J=1,N
0061        8 F(I,J)=F(I,J)+0.5*DT*C(I,J)+.16666667*DT*DT*S(I,J)
0062          CALL MATINV(F,N)
0063       25 IF(TM-TR+.5*DT)26,27,27
0064       26 IF(TM+DT-T(M2+1))10,10,11
0065       10 TM=TM+DT
0066          P=P-DG
0067          GO TO 12
0068       11 IF(M2-4)13,14,14
0069       13 M2=M2+1
0070          GO TO 15
0071       14 M2=1
0072          G(1)=G(5)
0073          T(1)=T(5)
0074          IS=IS+1
0075          READ33,ISC,(T(I),G(I),I=2,5)
0076          IF(IS-ISC)43,15,43
```

LIST L.P. 4

```
0077          15 TT=TM+DT-T(M2)                                                  CCC081
0078             TS=T(M2+1)-T(M2)                                                CCC082
0079             IF(TS)11,11,16                                                  CCC083
0080          16 GS=(G(M2+1)-G(M2))*386.4                                        CCC084
C081             GG=GS*TT/TS                                                     CCC085
C082             DG=GS*DT/TS                                                     CCC086
C083             P=-G(M2)*386.4-GG                                               CCC087
C084             TM=TM+DT                                                        CCC088
0085          12 DO17I=1,N                                                       CCC089
0086             AA(I)=VEL(I)+.5*DT*ACC(I)                                       CCCC90
C087          17 BB(I)=X(I)+DT*VEL(I)+.33333333*DT*DT*ACC(I)                     CCCC91
0088             CALL VECMTP(Y,S,BB,N,N)                                         CCCC92
0089             CALL VECMTP(Z,C,AA,N,N)                                         CCCC93
0090             DO18I=1,N                                                       CCCC94
0091          18 Y(I)=QM(I)*P-Z(I)-Y(I)                                          CCCC95
0092             CALL VECMTP(ACC,F,Y,N,N)                                        CCCC96
0093             DO19I=1,N                                                       CCCC97
0094             VEL(I)=AA(I)+.5*DT*ACC(I)                                       CCCC98
C095          19 X(I)=BB(I)+.16666667*DT*DT*ACC(I)                               CCCC99
0096             KC=KC+1                                                         CCC1C0
0097             IF(KC.LT.KP) GC TC 20                                           CCC1C1
0098             PRINT143,TM,(X(I),I=1,N)                                        CCC1C2
0099             KC=0                                                            CCC1C3
01C0          2C DO22I=1,N                                                       CCC1C4
0101             IF(ABS(X(I))-ABS(BX(I)))22,22,23                                CCC1C5
0102          23 BX(I)=X(I)                                                      CCC1C6
01C3             TX(I)=TM                                                        CCC1C7
0104          22 CONTINUE                                                        CCC1C8
01C5             GO TO 25                                                        CCC1C9
01C6          27 WRITE(6,34)
C107             WRITE(6,56) (I,BX(I),TX(I),I=1,N)
C108          56 FORMAT('0 MASS',I3,'   MAX DISPLACEMENT=',E13.4,'   AT TIME',E13.4)
                1)
0109             STCP                                                            CCC111
0110          43 PRINT35,IS                                                      CCC112
0111          35 FORMAT(' CARDS OUT OF ORDER',I5)                                CCC113
0112             STCP 3333                                                       CCC114
C113             END                                                             CCC115
```

```
                 C
C001                  SUBROUTINE VECMTP(B,A,Y,M,N)
                 C    (B)=(A)*(Y)   (A) OF ORDER MXN
C002                  DIMENSION A(20,20),Y(20),B(20)
C003                  DO1I=1,M
C004                  T=C.
C005                  DO2J=1,N
0006               2  T=T+A(I,J)*Y(J)
0007               1  B(I)=T
C008                  RETURN
0009                  END
```

LIST L.P. 4 (CONTINUED)

```
C0C1            SUBROUTINE MATINV(S,N)
C       ******************************************************************
C
C       INVERT MATRIX (S) IN SITU. (ORDER N X N)
C       USING JORDAN ELIMINATION METHOD
C
C       ******************************************************************
C0C2            DIMENSION S(20,20)
0003            DO 50 I=1,N
0004            DIAG=S(I,I)
C005            IF(DIAG.NE.0.0) GO TO 45
0006            WRITE(6,43)
0007         43 FORMAT(1HO,28HSTIFFNESS MATRIX IS SINGULAR//
                11X,16HHARD LUCK,BUDDY.)
0008            STOP
C
0009         45 S(I,I)=1.0
C010            DO 46 J=1,N
0011         46 S(I,J)=S(I,J)/DIAG
0012            DO 50 K=1,N
0013            IF(K.EQ.I) GO TO 50
0014            DIAG=S(K,I)
C015            S(K,I)=0.0
C016            DO 47 J=1,N
0017         47 S(K,J)=S(K,J)-DIAG*S(I,J)
C018         50 CONTINUE
C019            RETURN
C020            END
C
C0C1            SUBROUTINE TITLE
C
C       ******************************************************************
C
C       SUBROUTINE TO PRINT TITLE IN ASTERISKS
C
C       TITLE DATA CARD MUST HAVE A NUMBER GREATER THAN ZERO IN THE LAST
C       COLUMN IF TITLE CONTINUES ON TO THE FOLLOWING DATA CARD
C
C       ******************************************************************
C
C0C2            DIMENSION HEAD(19)
0003            WRITE(6,1)
0004          1 FORMAT(1H1,23X,82(1H*)/24X,1H*,80X,1H*)
C005          2 READ(5,3) HEAD,REST,KONT
0006          3 FORMAT(19A4,A3,I1)
0007            WRITE(6,4)(HEAD(I),I=1,19),REST
0008          4 FORMAT(1H ,23X,1H*,19A4,A3,1X,1H*/24X,1H*,80X,1H*)
0009            IF(KONT.GT.0) GO TO 2
C
C010            WRITE(6,5)
C011          5 FORMAT(1H ,23X,82(1H*)///)
C
C012            RETURN
0013            END
```

```
INPUT DATA ECHO CHECK

   3    1    4   0.7813E-02   0.15C0E 02

 EARTHQUAKE BEING USED IS THE EL CENTC MAY 1940 CNE.(NORTH-SCUTH).........

                          LATERAL STIFFNESS MATRIX

  0.4243E 06  -0.2589E 06   0.4530E 04
 -0.2589E 06   0.5725E 06  -0.3174E 06
  0.4530E 04  -0.3174E 06   0.3143E 06

                                DAMPING MATRIX

  0.C          0.0          0.0
  0.C          0.0          0.C
  0.0          0.0          0.C

                                                MASSES

  0.1369E 05   0.1294E 05   0.9310E 06

                ************************************************

                OUTPUT

 TIME                                  DISPLACEMENTS OF MASSES

 0.3125E-01  -0.1109E-02  -0.1478E-02  -0.1584E-02
 0.6250E-01  -0.2051E-02  -0.35C3E-02  -0.4781E-02
 0.9375E-01  -0.5277E-02  -0.83C3E-02  -0.1040E-01
 0.1250E 00  -0.9895E-02  -0.16C8E-01  -0.2064E-01

 0.1491E 02   0.1069E 01   0.1799E 01   0.2373E 01
 0.1494E 02   0.9792E 00   0.1619E 01   0.2106E 01
 0.1497E 02   0.7852E 00   0.1334E 01   0.1775E 01
 0.1500E 02   0.6686E 00   0.1097E 01   0.141RE 01

MASS  1   MAX DISPLACEMENT=   C.2870E 01   AT TIME   0.4539E 01

MASS  2   MAX DISPLACEMENT=   C.4793E 01   AT TIME   0.4547E 01

MASS  3   MAX DISPLACEMENT=   0.6291E 01   AT TIME   0.4547E 01
  STOP      0
/E
```

LIST L.P. 4 (CONTINUED)

APPENDIX V

SUMMARY OF CALCULATIONS UNDERTAKEN IN MODELLING THE THREE STOREY FRAME SHOWN IN FIGURE 6.1

Structural Member Sizes

Column — 22 ins internal diameter. Wall thicknesses 5/16 in 3/8 in and 7/16 in (top to bottom).

Rods — Circular section mild steel, diameters $2\frac{3}{8}$ in $2\frac{1}{8}$ in and $1\frac{3}{4}$ in (top to bottom).

Struts — Channel Sections (See Figure 6.1) 8 in 11.5 lb/ft, 9 in 13.4 lb/ft and 10 in 15.3 lb/ft.

Derived Member Properties

Columns — 5/16 in thick section

Cross sectional area	11 in^2
Second Moment of area	673 in^4
Total weight/column	1275 lb

3/8 in thick section

Cross sectional area	13.4 in^2
Second Moment of area	820 in^4
Total weight/column	1590 lb

7/16 in thick section

Cross sectional area	15.3 in^2
Second Moment of area	948 in^4
Total weight/column	1880 lb

Rods

2.375 in diameter

Cross sectional area	4.43 in^2
Length	509 in
Total weight/rod	640 lb

2.125 in diameter

Cross sectional area	3.55 in^2
Length	530 in
Total weight/rod	533 lb

1.75 in diameter

Cross sectional area	2.41 in^2
Length	566 in
Total weight/rod	386 lb

Struts

Upper Strut

Total weight	630 lb
Second Moment of area	86.6 in^4

Lower Strut

Total weight	786 lb
Second Moment of area	118.0 in^4

Self Weight Distribution

Assuming the self weight of the tower is lumped at the ground, first strut, second strut and ring girder levels respectively,

Weight (lbs) at ground	5,304
first strut	13,688
second strut	12,942
ring girder	5,110

treating the full structure as a plane frame.

Geometry

Lengths of braces		Angle to horizontal
Bottom bay	566 in	53.367°
Middle bay	530 in	52.18°
Top bay	509 in	49.734°

Horizontal member lengths:

Lower strut	344 in
Upper strut	304 in
Ring girder	264 in

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